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НАБЛИЖЕНИЙ РОЗВ"ЯЗОК ПЕРШОЇ ЗМІШАНОЇ ЗАДАЧІ ДЛЯ ЛІНІЙНОЇ СИСТЕМИ РІВНЯНЬ ПАРАБОЛІЧНОГО ТИПУ

За схемою* знайдено наближений розв"язок першої змішаної задачі для системи двох лінійних параболічних рівнянь зі сталими коефіцієнтами

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + B\frac{\partial u}{\partial x} + Au - f(x,t),\ 0<x<x_0,\ t>0 \qquad /1/$$

з початковими

$$u\Big|_{t=0} = \varphi(x) \qquad /2/$$

та крайовими

$$u\Big|_{x=0} = \mu(t);\quad u\Big|_{x=x_0} = \nu(t) \qquad /3/$$

умовами.

Тут $D=\|d_{ij}\|;\ B=\|b_{ij}\|;\ A=\|a_{ij}\|;\ i,j=1,2;\ f(x,t),\ \varphi(x),\ \mu(t),\ \nu(t)$ – задані двокомпонентні вектор-функції своїх аргументів.

На відрізках прямих $0<x<x_0,\ t=k\tau,\ k=1,2,\dots$ наближений розв"язок $u(x,k\tau) = u_k(x)$ задачі /1/ – /3/ зображається формулами

$$u_k(x) = v_k(x) + w_k(x),$$

в яких

$$v_k(x) = \int_0^{x_0} G(x,\xi)\Big[F_k(\xi) - \frac{v_{k-1}(\xi)}{\tau}\Big]d\xi,$$

$$v_0(x) = \varphi(x) - \big(\nu(0)-\mu(0)\big)\frac{x}{x_0} - \mu(0),$$

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* К о с т е н к о В.Г., Г у б а л ь Л.О. Наближений розв"язок змішаної задачі для параболічних систем рівнянь // Вісн.Львів. ун-ту. Сер.мех.-мат. 1986. Вип. 25. С. 28-31.

$$W_k(x) = (\vartheta_k - \mu_k)\frac{x}{x_0} + \mu_k,$$

$$F_k(\xi) = f_k(\xi) + (\vartheta'_k - \mu'_k)\frac{\xi}{x_0} + \mu'_k - B(\vartheta_k - \mu_k)\frac{1}{x_0} -$$

$$- A\left[(\vartheta_k - \mu_k)\frac{\xi}{x_0} + \mu_k\right],$$

елементи $G_{1m}(x,\xi)$ $G_{2m}(x,\xi)$ m -го стовпця $(m=1,2)$ матриці Гріна $G(x,\xi)$ мають вигляд

$$G_{1m}(x,\xi) = \frac{2}{\Delta\Delta^{(m)}}\Big\{(\gamma_{24} - \gamma_{23})e^{(\alpha_3+\alpha_4)x_0}\Big[\sum_{j=3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{2j} - \gamma_{22})e^{\alpha_1 x} +$$

$$+ (\gamma_{21} - \gamma_{2j})e^{\alpha_2 x}] + (\Delta_3^{(m)} e^{\alpha_3(x-\xi)} + \Delta_4^{(m)} e^{\alpha_4(x-\xi)})(\gamma_{22} - \gamma_{21})\Big] +$$

$$+ (\gamma_{24} - \gamma_{22})e^{(\alpha_2+\alpha_4)x_0}\Big[\sum_{j=2,4} e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{23} - \gamma_{2j})e^{\alpha_1 x} + (\gamma_{2j} - \gamma_{21})e^{\alpha_3 x}] +$$

$$+ (\Delta_2^{(m)} e^{\alpha_2(x-\xi)} + \Delta_4^{(m)} e^{\alpha_4(x-\xi)})(\gamma_{21} - \gamma_{23})\Big] +$$

$$+ (\gamma_{23} - \gamma_{22})e^{(\alpha_2+\alpha_3)x_0}\Big[\sum_{j=2,3} e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{2j} - \gamma_{24})e^{\alpha_1 x} + (\gamma_{21} - \gamma_{2j})e^{\alpha_4 x}] +$$

$$+ (\Delta_2^{(m)} e^{\alpha_2(x-\xi)} + \Delta_3^{(m)} e^{\alpha_3(x-\xi)})(\gamma_{24} - \gamma_{21})\Big] +$$

$$+ (\gamma_{24} - \gamma_{21})e^{(\alpha_1+\alpha_4)x_0}\Big[\sum_{j=1,4} e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{2j} - \gamma_{23})e^{\alpha_2 x} + (\gamma_{22} - \gamma_{2j})e^{\alpha_3 x}] +$$

$$+ (\Delta_1^{(m)} e^{\alpha_1(x-\xi)} + \Delta_4^{(m)} e^{\alpha_4(x-\xi)})(\gamma_{23} - \gamma_{22})\Big] +$$

$$+ (\gamma_{23} - \gamma_{21})e^{(\alpha_1+\alpha_3)x_0}\Big[\sum_{j=1,3} e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{24} - \gamma_{2j})e^{\alpha_2 x} + (\gamma_{2j} - \gamma_{22})e^{\alpha_4 x}] +$$

$$+ (\Delta_1^{(m)} e^{\alpha_1(x-\xi)} + \Delta_3^{(m)} e^{\alpha_3(x-\xi)})(\gamma_{22} - \gamma_{24})\Big] +$$

$$+ (\gamma_{22}-\gamma_{21}) e^{(\alpha_1+\alpha_2)x_0}\Big[\sum_{j=1,2} e^{-\alpha_j \xi}\Delta_j^{(m)}\big[(\gamma_{2j}-\gamma_{24})e^{\alpha_3 x}+(\gamma_{23}-\gamma_{2j})e^{\alpha_4 x}\big]+$$

$$+ (\Delta_1^{(m)} e^{\alpha_1(x-\xi)} + \Delta_2^{(m)} e^{\alpha_2(x-\xi)})(\gamma_{24}-\gamma_{23})\Big]\Big\}$$

для $0 \leq x \leq \xi$;

$$G_{1m}(x,\xi) = \frac{2}{\Delta\Delta^{(1)}}\Big\{(\gamma_{24}-\gamma_{23}) e^{(\alpha_3+\alpha_4)x_0}\Big[\sum_{j=1,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{22})e^{\alpha_1 x}+$$

$$+\sum_{j=2,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{21}-\gamma_{2j})e^{\alpha_2 x}\Big]+(\gamma_{24}-\gamma_{22})e^{(\alpha_2+\alpha_4)x_0}\Big[\sum_{j=1,2,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{23}-\gamma_{2j})e^{\alpha_1 x}+$$

$$+\sum_{j=2,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{21})e^{\alpha_3 x}\Big]+(\gamma_{23}-\gamma_{22})e^{(\alpha_2+\alpha_3)x_0}\Big[\sum_{j=1,2,3} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{24})e^{\alpha_1 x}+$$

$$+\sum_{j=2,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{21}-\gamma_{2j})e^{\alpha_4 x}\Big]+(\gamma_{24}-\gamma_{21})e^{(\alpha_1+\alpha_4)x_0}\Big[\sum_{j=1,2,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{23})e^{\alpha_2 x}+\sum_{j=1,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{22}-\gamma_{2j})e^{\alpha_3 x}\Big]+$$

$$+ (\gamma_{23}-\gamma_{21})e^{(\alpha_1+\alpha_3)x_0}\Big[\sum_{j=1,2,3} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{24}-\gamma_{2j})e^{\alpha_2 x}+\sum_{j=1,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{22})e^{\alpha_4 x}\Big]+$$

$$+ (\gamma_{22}-\gamma_{21})e^{(\alpha_1+\alpha_2)x_0}\Big[\sum_{j=1,2,3} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{24})e^{\alpha_3 x}+\sum_{j=1,2,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{23}-\gamma_{2j})e^{\alpha_4 x}\Big]\Big\}$$

для $\xi \leq x \leq x_0$;

$$G_{2m}(x,\xi) = \frac{2}{\Delta\Delta^{(1)}}\Big\{(\gamma_{24}-\gamma_{23})e^{(\alpha_3+\alpha_4)x_0}\Big[\sum_{j=3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}\big[(\gamma_{2j}-\gamma_{22})\gamma_{21}e^{\alpha_1 x}+(\gamma_{21}-\gamma_{2j})\gamma_{22}e^{\alpha_2 x}\big]+$$

$$+ (\Delta_3^{(m)} e^{\alpha_3(x-\xi)}\gamma_{23} + \Delta_4^{(m)} e^{\alpha_4(x-\xi)}\gamma_{24})(\gamma_{22}-\gamma_{21})\Big]+$$

$$+ (\gamma_{24}-\gamma_{22})e^{(\alpha_2+\alpha_4)x_0}\Big[\sum_{j=2,4} e^{-\alpha_j\xi}\Delta_j^{(m)}\big[(\gamma_{23}-\gamma_{2j})\gamma_{21}e^{\alpha_1 x}+(\gamma_{2j}-\gamma_{21})\gamma_{23}e^{\alpha_3 x}\big]+$$

$$+ (\Delta_2^{(m)} e^{\alpha_2(x-\xi)}\gamma_{22} + \Delta_4^{(m)} e^{\alpha_4(x-\xi)}\gamma_{24})(\gamma_{21}-\gamma_{23})\Big]+$$

$$+(\gamma_{23}-\gamma_{22})e^{(\alpha_2+\alpha_3)x_0}\Big[\sum_{j=2,3}e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{2j}-\gamma_{24})\gamma_{21}e^{\alpha_1 x}+(\gamma_{21}-\gamma_{2j})\gamma_{24}e^{\alpha_4 x}]+$$

$$+(\Delta_2^{(m)}e^{\alpha_2(x-\xi)}\gamma_{22}+\Delta_3^{(m)}e^{\alpha_3(x-\xi)}\gamma_{23})(\gamma_{24}-\gamma_{21})\Big]+$$

$$+(\gamma_{24}-\gamma_{21})e^{(\alpha_1+\alpha_4)x_0}\Big[\sum_{j=1,4}e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{2j}-\gamma_{23})\gamma_{22}e^{\alpha_2 x}+(\gamma_{22}-\gamma_{2j})\gamma_{23}e^{\alpha_3 x}]+$$

$$+(\Delta_1^{(m)}e^{\alpha_1(x-\xi)}\gamma_{21}+\Delta_4^{(m)}e^{\alpha_4(x-\xi)}\gamma_{24})(\gamma_{23}-\gamma_{22})\Big]+$$

$$+(\gamma_{23}-\gamma_{21})e^{(\alpha_1+\alpha_3)x_0}\Big[\sum_{j=1,3}e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{24}-\gamma_{2j})\gamma_{22}e^{\alpha_2 x}+(\gamma_{2j}-\gamma_{22})\gamma_{24}e^{\alpha_4 x}]+$$

$$+(\Delta_1^{(m)}e^{\alpha_1(x-\xi)}\gamma_{21}+\Delta_3^{(m)}e^{\alpha_3(x-\xi)}\gamma_{23})(\gamma_{22}-\gamma_{24})\Big]+$$

$$+(\gamma_{22}-\gamma_{21})e^{(\alpha_1+\alpha_2)x_0}\Big[\sum_{j=1,2}e^{-\alpha_j\xi}\Delta_j^{(m)}[(\gamma_{2j}-\gamma_{24})\gamma_{23}e^{\alpha_3 x}+(\gamma_{23}-\gamma_{2j})\gamma_{24}e^{\alpha_4 x}]+$$

$$+(\Delta_1^{(m)}e^{\alpha_1(x-\xi)}\gamma_{21}+\Delta_2^{(m)}e^{\alpha_2(x-\xi)}\gamma_{22})(\gamma_{24}-\gamma_{23})\Big]\Big\}$$

для $0\le x\le\xi$;

$$C_{2m}(x,\xi)=\frac{2}{\Delta\Delta^{(1)}}\Big\{(\gamma_{24}-\gamma_{23})e^{(\alpha_3+\alpha_4)x_0}\Big[\sum_{j=1,3,4}e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{22})\gamma_{21}e^{\alpha_1 x}+$$

$$+\sum_{j=2,3,4}e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{21}-\gamma_{2j})\gamma_{22}e^{\alpha_2 x}\Big]+(\gamma_{24}-\gamma_{22})e^{(\alpha_2+\alpha_4)x_0}\Big[\sum_{j=1,2,4}e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{23}-\gamma_{2j})\gamma_{21}e^{\alpha_1 x}+$$

$$+\sum_{j=2,3,4}e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{21})\gamma_{23}e^{\alpha_3 x}\Big]+(\gamma_{23}-\gamma_{22})e^{(\alpha_2+\alpha_3)x_0}\Big[\sum_{j=1,2,3}e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{24})\gamma_{21}e^{\alpha_1 x}+$$

$$+\sum_{j=2,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{21}-\gamma_{2j})\gamma_{24}e^{\alpha_4 x}]+(\gamma_{24}-\gamma_{21})e^{(\alpha_1+\alpha_4)x_0}[\sum_{j=1,2,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{23})\gamma_{22}e^{\alpha_2 x}+$$

$$+\sum_{j=1,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{22}-\gamma_{2j})\gamma_{23}e^{\alpha_3 x}]+(\gamma_{23}-\gamma_{21})e^{(\alpha_1+\alpha_3)x_0}[\sum_{j=1,2,3} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{24}-\gamma_{2j})\gamma_{22}e^{\alpha_2 x}+$$

$$+\sum_{j=1,3,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{22})\gamma_{24}e^{\alpha_4 x}]+(\gamma_{22}-\gamma_{21})e^{(\alpha_1+\alpha_2)x_0}[\sum_{j=1,2,3} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{2j}-\gamma_{24})\gamma_{23}e^{\alpha_3 x}+$$

$$+\sum_{j=1,2,4} e^{-\alpha_j\xi}\Delta_j^{(m)}(\gamma_{23}-\gamma_{2j})\gamma_{24}e^{\alpha_4 x}]\}$$

для $\xi \le x \le x_0$;

$\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$ – дійсні прості корені рівняння

$$\det[D\alpha^2+B\alpha+(A-\frac{1}{\tau}E)]=0,$$

$$\gamma_{2j}=\frac{d_{11}\alpha_j^2+b_{11}\alpha_j+a_{11}-\frac{1}{\tau}}{d_{12}\alpha_j^2+b_{12}\alpha_j+a_{12}},\quad j=\overline{1,4}$$

$$\Delta=\begin{vmatrix}1&1&1&1\\ \gamma_{21}&\gamma_{22}&\gamma_{23}&\gamma_{24}\\ e^{\alpha_1x_0}&e^{\alpha_2x_0}&e^{\alpha_3x_0}&e^{\alpha_4x_0}\\ \gamma_{21}e^{\alpha_1x_0}&\gamma_{22}e^{\alpha_2x_0}&\gamma_{23}e^{\alpha_3x_0}&\gamma_{24}e^{\alpha_4x_0}\end{vmatrix},\quad \Delta^{(1)}=\begin{vmatrix}1&1&1&1\\ \gamma_{21}&\gamma_{22}&\gamma_{23}&\gamma_{24}\\ \alpha_1&\alpha_2&\alpha_3&\alpha_4\\ \alpha_1\gamma_{21}&\alpha_2\gamma_{22}&\alpha_3\gamma_{23}&\alpha_4\gamma_{24}\end{vmatrix},$$

$$\Delta_1^{(1)}=\begin{vmatrix}0&\gamma_{22}-\gamma_{24}&\gamma_{23}-\gamma_{24}\\ \frac{d_{22}}{2\det D}&\alpha_2-\alpha_4&\alpha_3-\alpha_4\\ -\frac{d_{21}}{2\det D}&\alpha_2\gamma_{22}-\alpha_4\gamma_{24}&\alpha_3\gamma_{23}-\alpha_4\gamma_{24}\end{vmatrix},\quad \Delta_2^{(1)}=\begin{vmatrix}\gamma_{21}-\gamma_{24}&0&\gamma_{23}-\gamma_{24}\\ \alpha_1-\alpha_4&\frac{d_{22}}{2\det D}&\alpha_3-\alpha_4\\ \alpha_1\gamma_{21}-\alpha_4\gamma_{24}&\frac{-d_{21}}{2\det D}&\alpha_3\gamma_{23}-\alpha_4\gamma_{24}\end{vmatrix},$$

$$\Delta_3^{(1)}=\begin{vmatrix}\gamma_{21}-\gamma_{24}&\gamma_{22}-\gamma_{24}&0\\ \alpha_1-\alpha_4&\alpha_2-\alpha_4&\frac{d_{22}}{2\det D}\\ \alpha_1\gamma_{21}-\alpha_4\gamma_{24}&\alpha_2\gamma_{22}-\alpha_4\gamma_{24}&\frac{-d_{21}}{2\det D}\end{vmatrix},\quad \Delta_4^{(1)}=\begin{vmatrix}\gamma_{21}-\gamma_{22}&\gamma_{23}-\gamma_{22}&0\\ \alpha_1-\alpha_2&\alpha_3-\alpha_2&\frac{d_{22}}{2\det D}\\ \alpha_1\gamma_{21}-\alpha_2\gamma_{22}&\alpha_3\gamma_{23}-\alpha_2\gamma_{22}&\frac{-d_{21}}{2\det D}\end{vmatrix};$$

$\Delta_j^{(2)}$ одержуємо з $\Delta_j^{(1)}$, $j=\overline{1,4}$ заміною в останньому стовпця з елементами $0, \frac{d_{22}}{2\det D}, \frac{-d_{21}}{2\det D}$ відповідно стовпцем з елементами $0, \frac{-d_{12}}{2\det D}, \frac{d_{11}}{2\det D}$.

Зауважимо, що при цьому задовольняються рівності

$$\sum_{j=1}^{4}\Delta_j^{(1)}=0,\ \sum_{j=1}^{4}\gamma_{2j}\Delta_j^{(1)}=0,\ \sum_{j=1}^{4}\alpha_j\Delta_j^{(1)}=\frac{-d_{22}}{2\det D}\Delta,\ \sum_{j=1}^{4}\alpha_j\gamma_{2j}\Delta_j^{(1)}=\frac{d_{21}}{2\det D}\Delta;$$

$$\sum_{j=1}^{4}\Delta_j^{(2)}=0,\ \sum_{j=1}^{4}\gamma_{2j}\Delta_j^{(2)}=0,\ \sum_{j=1}^{4}\alpha_j\Delta_j^{(2)}=\frac{d_{12}}{2\det D}\Delta^{(1)},\ \sum_{j=1}^{4}\alpha_j\gamma_{2j}\Delta_j^{(2)}=\frac{-d_{11}}{2\det D}\Delta^{(1)}.$$

Цей алгоритм ефективно реалізовано на ЕОМ на прикладі першої змішаної задачі для системи рівнянь тепловологопереносу, яка є в найбільш простому випадку частинним випадком системи /1/.

Стаття надійшла до редколегії 04.03.86

УДК 517.946

В.М.Кирилич

ПРО ОДНУ НЕЛОКАЛЬНУ ЗАДАЧУ ТИПУ ДАРБУ ДЛЯ СТРОГО ГІПЕРБОЛІЧНОГО РІВНЯННЯ ДОВІЛЬНОГО ПОРЯДКУ

Нехай G – криволінійний сектор у верхній півплощині $t>0$ площини XOt, обмежений кривими γ_0 і γ_{m+1}, заданими відповідно рівняннями $x=a_0(t)$ і $x=a_{m+1}(t)$, $m\geq 0$, $a_0(0)=a_{m+1}(0)=0$, $a_{m+1}(t)>a_0(t)$ для всіх $t>0$. Криві $\gamma_s: x=a_s(t)$, $s=\overline{0,m+1}$, всі $a_s\in C^1(R_+)$, $a_{s+1}(t)>a_s(t)$ для всіх $t>0$, $a_s(0)=0$ розбивають G на $m+1$ компоненту зв"язності G^s, $s=\overline{0,m}$, пронумеровані зліва направо.

При кожному $s=\overline{0,m}$ в G^s розглянемо строго гіперболічне рівняння порядку $n\geq 2$

$$A^s u \equiv \sum_{i=0}^{n} A_i^s(x,t,\partial_x,\partial_t)u^s(x,t)=f^s(x,t), \qquad /1/$$