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INITIAL–BOUNDARY VALUE PROBLEM FOR DEGENERATED LINEAR HYPERBOLIC SYSTEM OF THE FIRST ORDER

In this paper we consider the linear degenerated hyperbolic system of the first order. For such system we assume that initial data are unbounded on the area $\Omega = (0,a) \times (0,b)$. Some conditions are obtained for the uniqueness and existence of the solution of the initial-boundary value problem. The initial and initial-boundary value problems for the first order linear hyperbolic systems were investigated by different authors [1–5].

Let $Q_T = \Omega \times (0,T)$. We shall consider in this domain the hyperbolic system of the form

$$u_{it}(x,y,t) + \sum_{j=1}^{n} a_{ij}(x,y,t)u_{jx}(x,y,t) + \sum_{j=1}^{n} b_{ij}(x,y,t)u_{jy}(x,y,t) +$$

$$+ \sum_{j=1}^{n} c_{ij}(x,y,t)u_j(x,y,t) = f_i(x,y,t) \quad i = 1,...,n. \tag{1}$$

For this system we impose the following initial and boundary conditions:

$$u_i(x,y,0) = \Theta_i(x,y), \quad i = 1,...,n, \tag{2}$$

$$\lim_{x\to 0} x^\beta u_i(x,y,t) = 0, \tag{3.1}$$

$$u_i(x,0,t) = \sum_{j=1}^{n} \alpha_{ij} u_j(x,b,t). \tag{3.2}$$

For the solution of problem (1) – (3) we will use eigenfunctions of the following problems:

$$(x^\alpha Z)' = \lambda Z; \lim_{x\to 0} x^\beta Z = 0, \quad Z'(a) = 0; \tag{4}$$

and

$$Z_i'' = \mu Z_i; \quad Z_i(0) = \sum_{j=1}^{n} \alpha_{ij} Z_j(b), \quad Z_i'(b) = \sum_{j=1}^{n} \alpha_{ij} Z_j'(0). \tag{5}$$

It is easy to show that for $\beta \in (0, 1/2)$, $\alpha \in (1,2)$ the eigenfunctions of problem (4) satisfy the following properties:

$$\lim_{x\to 0} xZ^2 = 0; \tag{4.1}$$

$$\lim_{x\to 0} x^\alpha Z Z' = 0; \tag{4.2}$$

$$\lim_{x\to 0} x^{\alpha+1}(Z')^2 = 0. \tag{4.3}$$

It is known that the eigenfunctions of problem (5) form a complete system in the space $L^2(0,b)$. For the next consideration we introduce the following system of conditions:

$$\begin{aligned}
(A): \quad & \mathfrak{A}A(x,b,t) - A(x,0,t)\mathfrak{A} \leqslant 0; \quad A^t = A;\\
& (A(a,y,t)\xi,\xi) \geqslant 0 \quad \forall \xi \in R^n;\\
(B): \quad & B(x,b,t) - \mathfrak{A}^t B(x,0,t)\mathfrak{A} = 0; \quad B^t = B;\\
(C): \quad & \mathfrak{A}C(x,b,t) - C(x,0,t)\mathfrak{A} \leqslant 0;\\
(\mathfrak{A}): \quad & \mathfrak{A}\mathfrak{A}^t = I,
\end{aligned}$$

where $\mathfrak{A}$ is the matrix with coefficients α_{ij}.

Theorem. *If conditions $(A), (B), (C)$, $(\mathfrak{A})$ hold, $\beta \in (0, 1/2)$, $\alpha \in (1,2)$ and*

$$A_x, x^{-1}A, x^{-\alpha/2}A_y, B, x^{\alpha/2}B_x, B_y, C, x^{\alpha/2}C_x, C_y \in L^\infty(Q_T);$$
$$f, x^{\alpha/2}f_x \in L^2(Q_T); \quad \Theta_i, x^{\alpha/2}\Theta_{ix}, \Theta_{iy} \in L^2(\Omega)$$

and f satisfies (3.1), (3.2), Θ_i satisfies (3.2), then there exists a unique one solution $u = (u_1, \ldots, u_n)$ of problem (1) - (3) such that u, u_t, u_y, $x^{\alpha/2}u_x \in L^2(Q_T)$.

Proof. Let $\phi^i_m(x)$ be the eigenfunctions of problem (4) and $\psi^i_l(y)$ the eigenfunctions of problem (5). We will consider the sequence of the form

$$u^N_i(x,y,t) = \sum_{m=1}^{N}\sum_{l=1}^{N} \beta^N_{mli}(t)\phi^i_m(x)\psi^i_l(y),$$

where β^N_{mli}, $i = 1, \ldots, n$, $m, l = 1, \ldots, N$, are the solutions of the following Cauchy problem

$$\begin{aligned}
\int_{\Omega_\tau} [(u^N_{it}\phi^i_m(x)\psi^i_l(y) + \sum_{j=1}^{n} a_{ij}u_{jx}\phi^i_m(x)\psi^i_l(y) + \sum_{j=1}^{n} b_{ij}u^N_{jy}\phi^i_m(x)\psi^i_l(y) + \\
+ \sum_{j=1}^{n} c_{ij}u^N_j\phi^i_m(x)\psi^i_l(y) - f_i\phi^i_m(x)\psi^i_l(y)]dxdy = 0, \qquad (6)\\
\beta^N_{mli}(0) = \Theta^N_{mli}; \quad \text{for } m, l = 1, ..., N, \ i = 1, ..., n.
\end{aligned}$$

Here

$$\Theta^N_i = \sum_{m=1}^{N}\sum_{l=1}^{N} \Theta^N_{mli}\phi^i_m(x)\psi^i_l(y), \ \ \Theta^N_i \to \Theta_i;$$
$$x^{\alpha/2}\Theta^N_{ix} \to x^{\alpha/2}\Theta_{ix}; \ \Theta^N_{iy} \to \Theta_{iy} \text{ in } L^2(\Omega).$$

Multiplying every equations of the system (6) by the functions β_{mli} respectvely, summing from $l, m = 1$ to N and integrating on Q_τ we get

$$\begin{aligned}
\int_{Q_\tau} [(u^N_t, u^N) + (Au^N_x, u^N) + \\
+ (Bu^N_y, u^N) + (Cu^N, u^N) - (f, u^N)]dxdydt = 0 \text{ for } \tau \in (0,T].
\end{aligned}$$

Taking into account assumptions of the theorem we obtain

$$I_1 = \int\limits_{Q_\tau} (u_t^N, u^N)dxdydt = \frac{1}{2}\int\limits_{\Omega_{tau}} |u^N|^2 - \frac{1}{2}\int\limits_{\Omega_0} |\Theta^N|^2 dxdy$$

$$I_2 = \int\limits_{Q_\tau} (Au_x^N, u^N)dxdydt = \frac{1}{2}\int\limits_{Q_\tau} (Au^N, u^N)_x dxdydt - \frac{1}{2}\int\limits_{Q_\tau} (A_x u^N, u^N)dxdydt =$$

$$= \frac{1}{2}\int\limits_0^b\int\limits_0^\tau (Au^N, u^N)|_{x=a}dydt - \frac{1}{2}\int\limits_0^b\int\limits_0^\tau (Au^N, u^N)|_{x=0}dydt -$$

$$- \frac{1}{2}\int\limits_{Q_\tau} (A_x u^N, u^N)dxdydt \geqslant -\frac{a_1}{2}\int\limits_{Q_\tau} |u^N|^2 dxdydt$$

where a_1 depends on coefficients of the matrix A. Going on we get

$$I_3 = \int\limits_{Q_\tau} (Bu_y^N, u^N)dxdydt = \frac{1}{2}\int\limits_{Q_\tau} (Bu^N, u^N)_y dxdydt - \frac{1}{2}\int\limits_{Q_\tau} (B_y u^N, u^N)dxdydt =$$

$$= \frac{1}{2}\int\limits_0^a\int\limits_0^\tau (B(x,b,t)u^N, u^N)dxdt - \frac{1}{2}\int\limits_0^a\int\limits_0^\tau (B(x,0,t)u^N, u^N)dxdt -$$

$$- \frac{1}{2}\int\limits_{Q_\tau} (B_y u^N, u^N)dxdydt - \frac{1}{2}\int\limits_0^a\int\limits_0^\tau [(B(x,b,t)u^N, u^N) -$$

$$- (B(x,0,t)\mathfrak{A}u^N, \mathfrak{A}u^N)]dxdt - \frac{1}{2}\int\limits_{Q_\tau} (B_y u^N, u^N)dxdydt -$$

$$- \frac{1}{2}\int\limits_0^a\int\limits_0^\tau [(B(x,b,t)u^N, u^N) - (\mathfrak{A}^t B(x,0,t)\mathfrak{A}u^N, u^N)]dxdt -$$

$$- \frac{1}{2}\int\limits_{Q_\tau} (B_y u^N, u^N)dxdydt \geqslant -\frac{b_1}{2}\int\limits_{Q_\tau} |u^N|^2 dxdydt,$$

where b_1 depends on coefficients of the matrix B.

From the conditions of the theorem we obtain

$$I_4 = \int\limits_{Q_\tau} (Cu^N, u^N)dxdydt \geqslant c_0 \int\limits_{Q_\tau} |u^N|^2 dxdydt,$$

$$I_5 = \int\limits_{Q_\tau} (f, u^N)dxdydt \leqslant \frac{1}{2\delta_0}\int\limits_{Q_\tau} |f|^2 dxdydt + \frac{\delta_0}{2}\int\limits_{Q_\tau} |u^N|^2 dxdydt$$

for $\delta_0 > 0$. From the above estimates of integrals $I_1, \ldots, I_5$ the inequality follows

$$\int\limits_{\Omega_\tau} |u^N|^2 + \int\limits_{Q_\tau} M|u^N|^2 dxdydt \leqslant \frac{1}{\delta_0}\int\limits_{Q_\tau} |f|^2 dxdydt + \int\limits_{\Omega_0} |\Theta^N|^2 dxdy,$$

where $M = 2c_0 - a_1 - b_1 - \delta_0$. From here using the Gronoull-Bellman inequality we obtain

$$\int_{\Omega_\tau} |u^N|^2 dxdy \leqslant \mu_1(\tau). \tag{7}$$

Now using (4), (6) it is easy to get the following equality

$$\int_{Q_\tau} [(u_t^N, (x^\alpha u_x^N)_x) + (Au_x^N, (x^\alpha u_x^N)_x) + (Bu_y^N, (x^\alpha u_x^N)_x) + \tag{8}$$
$$+ (Cu^N, (x^\alpha u_x^N)_x) - (f, (x^\alpha u_x^N)_x)] dxdydt = 0, \quad \tau \in (0,T].$$

Hence using assumptions of the theorem we shall have

$$I_6 = \int_{Q_\tau} (u_t^N, (x^\alpha u_x^N)_x) dxdydt = \int_{Q_\tau} (u_t^N, x^\alpha u_x^N)_x dxdydt - \int_{Q_\tau} (u_{tx}^N, x^\alpha u_x^N) dxdydt =$$
$$= \int_0^\tau \int_0^b (u_t^N, x^\alpha u_x^N)|_{x=a} dydt - \int_0^\tau \int_0^b (u_t^N, x^\alpha u_x^N)|_{x=0} dydt -$$
$$- \frac{1}{2} \int_{Q_\tau} (x^{\alpha/2} u_x^N, x^{\alpha/2} u_x^N)_t dxdydt = -\frac{1}{2} \int_{\Omega_\tau} x^\alpha |u_x^N|^2 dxdt + \frac{1}{2} \int_{\Omega_0} x^\alpha |\Theta_x^N|^2 dxdt;$$

$$I_7 = \int_{Q_\tau} (Au_x^N, (x^\alpha u_x^N)_x) dxdydt = \int_{Q_\tau} ((Ax^{-\alpha})(x^\alpha u_x^N), (x^\alpha u_x^N)_x)) dxdydt =$$
$$= \frac{1}{2} \int_{Q_\tau} ((Ax^{-\alpha}) x^\alpha u_x^N, x^\alpha u_x^N)_x) dxdydt - \frac{1}{2} \int_{Q_\tau} ((Ax^{-\alpha})_x x^\alpha u_x^N, x^\alpha u_x^N)) dxdydt =$$
$$= -\frac{1}{2} \int_{Q_\tau} ((A_x - \alpha A x^{-1}) u_x^N, x^\alpha u_x^N) dxdydt \geqslant \frac{a_2}{2} \int_{Q_\tau} |x^\alpha u_x^N|^2 dxdydt,$$

where a_2 depends on coefficients of the matrix A_x

$$I_8 = \int_{Q_\tau} (Bu_y^N, (x^\alpha u_x^N)_x) dxdydt = \int_{Q_\tau} (Bu_y^N, x^\alpha u_x^N)_x dxdydt -$$
$$- \int_{Q_\tau} (B_x u_y^N, x^\alpha u_x^N) dxdydt - \int_{Q_\tau} (Bu_{xy}^N, x^\alpha u_x^N) dxdydt -$$
$$- \int_{Q_\tau} (B_x u_y^N, x^\alpha u_x^N) dxdydt - \frac{1}{2} \int_{Q_\tau} (Bu_x^N, x^\alpha u_x^N)_y dxdydt +$$
$$+ \frac{1}{2} \int_{Q_\tau} (B_y u_x^N, x^\alpha u_x^N) dxdydt = I_8^1 + I_8^2 + I_8^3,$$

$$I_8^1 = \int_{Q_\tau} (B_x u_y^N, x^\alpha u_x^N) dxdydt \leqslant \int_{Q_\tau} \|B_x\| |x^{\alpha/2} |u_y^N| |x^{\alpha/2} u_x^N| dxdydt \leqslant$$
$$\leqslant \frac{b_2}{2} \int_{Q_\tau} (|u_y^N|^2 + x^\alpha |u_x^N|^2) dxdydt,$$

where b_2 depends on coefficients of the matrix B_x;

$$I_8^2 = -\frac{1}{2}\int_0^a\int_0^\tau [(B(x,b,t)u_x^N(x,b,t), u_x^N(x,b,t))-$$

$$-(B(x,0,t)u_x^N(x,0,t), u_x^N(x,0,t))]x^\alpha dxdt =$$

$$= -\frac{1}{2}\int_0^a\int_0^\tau [(B(x,b,t)u_x^N(x,b,t), u_x^N(x,b,t))-$$

$$-(B(x,0,t)\mathfrak{A}u_x^N(x,b,t), \mathfrak{A}u_x^N(x,b,t))]x^\alpha dxdt \leqslant 0;$$

$$I_8^3 = \frac{1}{2}\int_{Q_\tau}(B_y u_x^N, x^\alpha u_x^N)dxdydt \leqslant b_3\int_{Q_\tau} x^\alpha |u_x^N|^2 dxdydt,$$

where b_3 depends on coefficients of the matrix B_y;

$$I_9 = \int_{Q_\tau}(Cu^N, (x^\alpha u_x^N)_x)dxdydt = \int_{Q_\tau}(Cu^N, x^\alpha u_x^N)_x dxdydt - \int_{Q_\tau}(C_x u^N, x^\alpha u_x^N)dxdydt-$$

$$-\int_{Q_\tau}(Cu_x^N, x^\alpha u_x^N)dxdydt = I_9^1 + I_9^2 + I_9^3; \quad I_9^1 = 0;$$

$$I_9^2 = \int_{Q_\tau}(C_x u^N, x^\alpha u_x^N)dxdydt \leqslant \int_{Q_\tau}\|C_x\||x^{\alpha/2}|u^N||x^{\alpha/2}u_x^N|dxdydt \leqslant$$

$$\leqslant \frac{c_2}{2\delta_1}\int_{Q_\tau}(|u^N|^2 dxdydt + \frac{\delta_1}{2}\int_{Q_\tau} x^\alpha |u_x^N|^2 dxdydt,$$

where $\delta_1 > 0$ and c_2 depend on coefficients of the matrix C_x;

$$I_9^3 = \int_{Q_\tau}(Cu_x^N, x^\alpha u_x^N)dxdydt \leqslant -c_0\int_{Q_\tau} x^\alpha |u_x^N|^2 dxdydt;$$

$$I_{10} = -\int_{Q_\tau}(f, (x^\alpha u_x^N)_x)dxdydt = \int_{Q_\tau}(f, x^\alpha u_x^N)_x dxdydt +$$

$$+\int_{Q_\tau}(f_x, x^\alpha u_x^N)dxdydt = I_{10}^1 + I_{10}^2; \quad I_{10}^1 = 0;$$

$$I_{10}^2 \leqslant \frac{1}{2\delta_2}\int_{Q_\tau}|f_x|^2 x^\alpha dxdydt + \frac{\delta_2}{2}\int_{Q_\tau} x^\alpha |u_x^N|^2 dxdydt.$$

Taking into account estimates of integrals $I_6, \dots, I_{10}$ we obtain from (8) the inequality

$$\int_{\Omega_\tau} x^\alpha |u_x^N|^2 + \int_{Q_\tau} M_1 x^\alpha |u_x^N|^2 dxdydt \leqslant \frac{1}{\delta_2}\int_{Q_\tau} x^\alpha |f_x|^2 dxdydt +$$

$$+\int_{\Omega_0} x^\alpha |\Theta_x^N|^2 dxdy + \frac{c_2}{\delta_1}\int_{Q_\tau}|u^N|^2 dxdydt + \frac{b_2}{2}\int_{Q_\tau}|u_y^N|^2 dxdydt,$$

where $M_1 = 2c_0 - 2\delta_1 - 2b_2 - a_2 - b_3$. Again using (5), (6) it is easy to get the following equality

$$\int_{Q_\tau} [(u_t^N, u_{yy}^N) + (Au_x^N, u_{yy}^N) + (Bu_y^N, u_{yy}^N) + \\ + (Cu^N, u_{yy}^N) - (f, u_{yy}^N)]dxdydt = 0, \quad \tau \in (0, T]. \tag{9}$$

Taking into account assumptions of the theorem we obtain

$$I_{11} = \int_{Q_\tau} (u_t^N, u_{yy}^N)dxdydt = \int_{Q_\tau} (u_t^N, u_y^N)_y dxdydt - \int_{Q_\tau} (u_{ty}^N, u_y^N)dxdydt =$$

$$= \int_{Q_\tau} (u_t^N, u_y^N)_y dxdydt - \frac{1}{2}\int_{Q_\tau} (u_t^N, u_y^N)_t dxdydt = I_{11}^1 + I_{11}^2;$$

$$I_{11}^1 = \int_0^a \int_0^\tau [(u_t^N(x,b,t), u_y^N(x,b,t)) - u_t^N(x,0,t), u_y^N(x,0,t))]dxdt =$$

$$= \int_0^a \int_0^\tau [(u_t^N(x,b,t), \mathfrak{A}u_y^N(x,0,t)) - (\mathfrak{A}u_t^N(x,b,t), u_y^N(x,0,t))]dxdt = 0;$$

$$I_{11} = I_{11}^2 = -\frac{1}{2}\int_{\Omega_\tau} x^\alpha |u_y^N|^2 dxdt + \frac{1}{2}\int_{\Omega_0} x^\alpha |\Theta_y^N|^2 dxdt;$$

$$I_{12} = \int_{Q_\tau} (Au_x^N, u_{yy}^N)dxdydt = \int_{Q_\tau} (Au_x^N, u_y^N)_y dxdydt - \int_{Q_\tau} (A_y u_x^N, u_y^N)dxdydt -$$

$$- \int_{Q_\tau} (Au_{xy}^N, u_y^N)dxdydt = I_{12}^1 + I_{12}^2 + I_{12}^3;$$

$$I_{12}^1 = \int_0^a \int_0^\tau [(A(x,b,t)u_x^N(x,b,t), u_y^N(x,b,t)) -$$

$$- (A(x,0,t)u_x^N(x,0,t), u_y^N(x,0,t))]dxdt =$$

$$= \int_0^a \int_0^\tau [(A(x,b,t)u_x^N(x,b,t), \mathfrak{A}u_y^N(x,0,t)) -$$

$$- (A(x,0,t)\mathfrak{A}u_x^N(x,b,t), u_y^N(x,0,t))]dxdt =$$

$$= \int_0^a \int_0^\tau [(\mathfrak{A}A(x,b,t) - A(x,0,t)\mathfrak{A})u_x^N(x,b,t)u_y^N(x,0,t)]dxdt \leqslant 0;$$

$$I_{12}^2 = -\int_{Q_\tau} (A_y u_x^N, u_y^N)dxdydt \leqslant \int_{Q_\tau} \|A_y\| x^{\alpha/2}|u_y^N| |x^{\alpha/2}u_x^N| dxdydt \leqslant$$

$$\leqslant \frac{a_3}{2}\int_{Q_\tau} (|u_y^N|^2 + x^\alpha |u_x^N|^2)dxdydt,$$

where a_3 depends on coefficients of the matrix A_y;

$$I_{12}^3 = -\int\limits_{Q_\tau} (Au_{xy}^N, u_y^N)dxdydt - \frac{1}{2}\int\limits_{Q_\tau} (Au_y^N, u_y^N)_x dxdydt + \\ + \frac{1}{2}\int\limits_{Q_\tau} (A_x u_y^N, u_y^N)dxdydt \leqslant \frac{a_{21}}{2}\int\limits_{Q_\tau} |u_y^N|^2 dxdydt;$$

where a_{21} depends on coefficients of the matrix A_x;

$$I_{13} = \int\limits_{Q_\tau} (Bu_y^N, u_{yy}^N)dxdydt = \frac{1}{2}\int\limits_{Q_\tau} (Bu_y^N, u_y^N)_y dxdydt - \\ - \frac{1}{2}\int\limits_{Q_\tau} (B_y u_y^N, u_y^N)dxdydt = I_{13}^1 + I_{13}^2;$$

$$I_{13}^1 = \frac{1}{2}\int\limits_0^a \int\limits_0^\tau [(B(x,b,t)u_y^N(x,b,t), u_y^N(x,b,t)) - \\ - (B(x,0,t)u_y^N(x,0,t), u_y^N(x,0,t))]dxdt = \\ = \frac{1}{2}\int\limits_0^a \int\limits_0^\tau [(B(x,b,t)\mathfrak{A}u_y^N(x,0,t), \mathfrak{A}u_y^N(x,0,t)) - \\ - (B(x,0,t)u_y^N(x,0,t), u_y^N(x,0,t))]dxdt = \\ = \frac{1}{2}\int\limits_0^a \int\limits_0^\tau [(\mathfrak{A}^t B(x,b,t)\mathfrak{A} - B(x,0,t))u_y^N(x,0,t)u_y^N(x,0,t))]dxdt = 0;$$

$$I_{13}^2 = \frac{1}{2}\int\limits_{Q_\tau} (B_y u_y^N, u_y^N)dxdydt \leqslant \frac{b_3}{2}\int\limits_{Q_\tau} |u_y^N|^2 dxdydt,$$

where b_3 depends on coefficients of the matrix B_y;

$$I_{14} = \int\limits_{Q_\tau} (Cu^N, u_{yy}^N)dxdydt = \int\limits_{Q_\tau} (Cu^N, u_y^N)_y dxdydt - \int\limits_{Q_\tau} (C_y u^N, u_y^N)dxdydt - \\ - \int\limits_{Q_\tau} (Cu_y^N, u_y^N)dxdydt = I_{14}^1 + I_{14}^2 + I_{14}^3;$$

$$I_{14}^1 = \int\limits_0^a \int\limits_0^\tau [(C(x,b,t)u^N(x,b,t), u_y^N(x,b,t)) - \\ - (C(x,0,t)u^N(x,0,t), u_y^N(x,0,t))]dxdt = \\ = \int\limits_0^a \int\limits_0^\tau [(C(x,b,t)u^N(x,b,t), \mathfrak{A}u_y^N(x,0,t)) - \\ - (C(x,0,t)\mathfrak{A}u^N(x,b,t), u_y^N(x,0,t))]dxdt \leqslant 0;$$

$$I_{14}^2 = -\int_{Q_\tau} (C_y u^N, u_y^N) dxdydt \leqslant \frac{c_3}{2\delta_3} \int_{Q_\tau} |u^N|^2 dxdydt + \frac{\delta_3}{2} \int_{Q_\tau} |u_y^N|^2) dxdydt,$$

where c_3 depends on coefficients of the matrix C_y;

$$I_{14}^3 \leqslant -c_0 \int_{Q_\tau} |u_y^N|^2 dxdydt; \quad I_{15} = -\int_{Q_\tau} (f, u_{yy}^N) dxdydt =$$

$$= -\int_{Q_\tau} (f, u_y^N)_y dxdydt + \int_{Q_\tau} (f_y, u_y^N) dxdydt = I_{15}^1 + I_{15}^2;$$

$$I_{15}^1 = -\int_0^a \int_0^\tau [(f(x,b,t), u_y^N(x,b,t)) - (f(x,0,t), u_y^N(x,0,t))] dxdt =$$

$$= \int_0^a \int_0^\tau [((\mathfrak{A} f(x,b,t) - f(x,0,t)), u_y^N(x,0,t))] dxdt = 0;$$

$$I_{15}^2 \leqslant \frac{1}{2\delta_4} \int_{Q_\tau} |f_y|^2 dxdydt + \frac{\delta_4}{2} \int_{Q_\tau} |u_y^N|^2 dxdydt.$$

From the estimates of integrals $I_{11}, \ldots, I_{15}$ and equality (9) we obtain

$$\int_{\Omega_\tau} |u_y^N|^2 + \int_{Q_\tau} M_2 |u_y^N|^2 dxdydt \leqslant \frac{1}{\delta_4} \int_{Q_\tau} x^\alpha |f_y|^2 dxdydt + \tag{10}$$

$$+ \int_{\Omega_0} |\Theta_y^N|^2 dxdy + \frac{c_3}{\delta_3} \int_{Q_\tau} |u^N|^2 dxdydt + a_3 \int_{Q_\tau} x^\alpha |u_x^N|^2 dxdydt,$$

where $M_2 = 2c_0 - 2\delta_4 - 2b_3 - a_{21} - a_3$. If we sum (9) and (10) we get

$$\int_{\Omega_\tau} (x^\alpha |u_x^N|^2 + |u_y^N|^2) dxdy \leqslant \mu \int_{\Omega_0} (x^\alpha |u_x^N|^2 + |u_y^N|^2) dxdydt + f_0(\tau). \tag{11}$$

Then from (11) and the Gronoull-Bellman inequality we obtain the estimate

$$\int_{Q_\tau} \sum_{i=1}^n x^\alpha |u_{ix}^N|^2 dxdydt + \int_{Q_\tau} \sum_{i=1}^n |u_{iy}^N|^2 dxdydt \leqslant \|\Theta_x\|_{L_\alpha^2(\Omega)} + \frac{1}{\delta_3} \|f_x\|_{L_\alpha^2(\Omega)} +$$

$$+ \|\Theta_y\|_{L^2(\Omega)} + \|f_y\|_{L^2(\Omega)} - \mu_1(\tau) - \mu_2(\tau) \leqslant D(\tau).$$

Therefore

$$\|u^N\|_{L^2(\Omega)} + \|x^{\alpha/2} u_x^N\|_{L_\alpha^2(\Omega)} + \|u_y^N\|_{L^2(\Omega)} \leqslant D(\tau).$$

And now we can choose from sequences $\{u_i^N(x,y,t)\}$, $\{u_{ix}^N(x,y,t)\}$, $\{u_{iy}^N(x,y,t)\}$ subsequences such that $u_i^m(x,y,t) \to u_i(x,y,t)$ weakly in $L^2(\Omega)$, $x^{\alpha/2} u_{ix}^m(x,y,t) \to x^{\alpha/2} u_{ix}(x,y,t)$ weakly in $L^2(\Omega)$, $u_{iy}^m(x,y,t) \to u_{iy}(x,y,t)$ weakly in $L^2(\Omega)$.

Now it is easy to show that the function $u(x,y,t)$ is the unique solution of problem (1)–(3).

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ПОЧАТКОВО-КРАЙОВА ЗАДАЧА ДЛЯ ЛІНІЙНОЇ ГІПЕРБОЛІЧНОЇ СИСТЕМИ ПЕРШОГО ПОРЯДКУ З ВИРОДЖЕННЯМ

У праці досліджено мішану задачу для системи гіперболічних рівнянь першого порядку у випадку двох просторових змінних з виродженням на частині межі (за змінною x). За іншою змінною (змінною y) розглянуто нелокальні крайові умови. Початкові функції є необмежені в деякому околі межі. Отримано певні достатні умови існування та єдиності розв'язку зазначеної задачі.

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