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VIBRATION ANALYSIS OF BEAMS WITH CRACKS

1. Introduction. In many problems of dynamics structural elements are modelled by beams. Damages of the structural elements in form of cracks can appear. The size and localisation of cracks can essentially influence structural safety. By analysis of influence of size and localisation of crack on eigenfrequencies it is usually possible to modify the structure in such a way that the crack does not enlarge further [1]. A technique based on the sensitivity of the natural frequencies with respect to the size and positon of cracks in beams is thus presented in the paper. By assuming that a crack may be modelled by an effective elastic hinge [2], with the rotational inertia and shear deformations taken into account [3], the problem is formulated on the basis of the classic theory of Timoshenko's beams. The formulation is illustrated by a number of numerical results related to three prismatic beams with rectangular cross-sections.

2. Elastic hinge modelling. Assume that the cross-section $b{\times}h$ of a beam weakened by a crack of length (depth) a, Fig. 1, can effectively be replaced by an elastic hinge of stiffness K being a decreasing function of the crack length, $K = K(a)$.

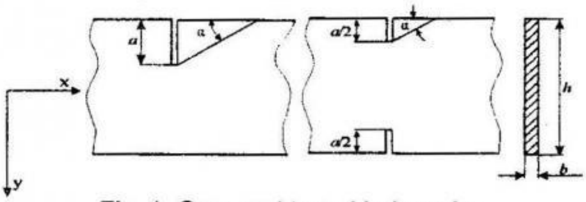

Fig. 1. One- and two-sided cracks.

The function can be determined either experimentally or analytically, basing on deformation hypothesis for one-sided cracks [2, pp. 75-76]

$$K(a) = E\,I_{yy}\,\frac{(h-a)^2}{a^2(3h-2a)}\,\mathrm{tg}\alpha \qquad (1)$$

and, for two-sided cracks as

$$K(a) = E\,I_{yy}\,\frac{2(h-a)^2}{a^2(3h-2a)}\,\mathrm{tg}\alpha \qquad (2)$$

with $I_{yy} = bh^3/12$ being the moment of inertia of the cross-section about the z-axis and E the Young modulus. The distribution of stresses around the cracks is taken into account in defining the angle a. For example, for one-sided cracks, [4]

$$\alpha = \mathrm{arctg}\left[\frac{1}{2}\left(\mathrm{tg}\,\frac{\pi}{6} + \frac{a}{h}\right)\right] \qquad (3)$$

and for two-sided cracks

$$\alpha = \operatorname{arctg}\left[\frac{1}{2}\left(\operatorname{tg}\frac{\pi}{6} + \frac{a}{2h}\right)\right]. \tag{4}$$

In the context of crack theory the plane-stress-state stiffness $K(a)$ can be obtained as

$$K(a) = \left[\ \frac{2b}{E}\int_0^a \left(K_{I_M}^0\right)^2 d\xi\ \right]^{-1}, \tag{5}$$

where $K_{I_M}^0$ is the stress intensity factor for unit internal bending moment at cross-section of the beam with the crack. For one-sided cracks, [5]

$$K_{I_M}^0 = \frac{6}{bh^2}\sqrt{\pi a}\,F(a/h), \tag{6}$$

where

$$F(a/h) = 1.122 - 1.4\,(a/h) + 7.33\,(a/h)^2 - 13.08\,(a/h)^3 + 14.0\,(a/h)^4 \tag{7}$$

and for two-sided ones

$$K_{I_M}^0 = \frac{6}{bh^2}\sqrt{\pi a}\,(1 - a/h)^{-3/2}\,G(a/h), \tag{8}$$

where

$$G\!\left(\frac{a}{h}\right) = \frac{4}{3\pi}\left[1 + \frac{1}{2}\left(1-\frac{a}{h}\right) + \frac{3}{8}\left(1-\frac{a}{h}\right)^2 + \frac{5}{16}\left(1-\frac{a}{h}\right)^3\right] - 0.47\left(1-\frac{a}{h}\right)^4 + 0.663\left(1-\frac{a}{h}\right)^5. \tag{9}$$

Values of elastic hinge stiffness for steel beams with $E = 2.06\times10^5\mathrm{Nmm}^{-2}$, $h = 40\mathrm{mm}$, $b = 7\mathrm{mm}$, are shown in Fig. 2. Agreement of the results obtained on the basis of beam theory as well as by fracture mechanics approach is quite good. That is why further the beam theory will be applied only.

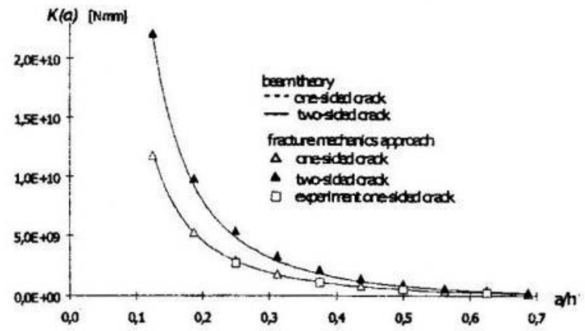

Fig. 2. Elastic hinge stiffness function.

At the crack position $x = x_0$ the conditions of: equilibrium (10), continuity of deflections (11) and bending moments (12) and shear forces (13), are to be satisfied, [6].

$$EI\,y''(x_0) = K(a)[y'(x_0^+) - y'(x_0^-)], \tag{10}$$

$$y(x_0^-) = y(x_0^+), \tag{11}$$

$$E\,I_{yy}\,y''(x_0^-) = E\,I_{yy}\,y''(x_0^+), \tag{12}$$

$$E\,I_{yy}\,y'''(x_0^-) = E\,I_{yy}\,y'''(x_0^+). \tag{13}$$

3. Numerical results. Starting with the equations of Timoshenko's beam vibrations [3, pp. 318-320], calculations were carried out for three types of the beam supports, depending on the crack position and depth, for one- and two-sided cracks. The two smallest natural frequencies values obtained were compared with those calculated by the finite element code NASTRAN, cf. [7] (the calculations were performed at TASK Academic Computer Centre using NASTRAN v70). Calculated natural frequencies $\varpi_n(\xi)$, $n = 1,2$, for beams with the cracks we compared to $\varpi_n(0)$, e.i. for corresponding beams without cracks. The beam was modelled using 960 CQUAD8 elements of 5×5mm, and 2 CRAC2D elements of 2×2mm from the NASTRAN element library. Values of the basic material parameters were as follows: Young modulus $E = 2.06\times10^5 \text{Nmm}^{-2}$, Poisson ratio $\nu = 0.33$, mass density $\rho = 7.83\times10^{-9}\ \text{Ns}^2\text{mm}^{-4}$.

Below some results of the calculations are presented

Case 1.

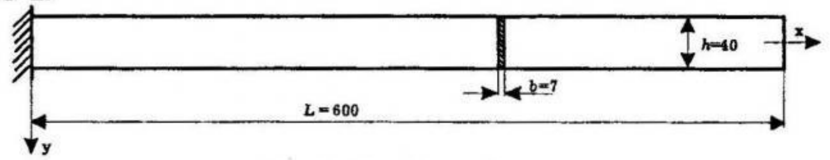

Fig. 3. Cantilever beam.

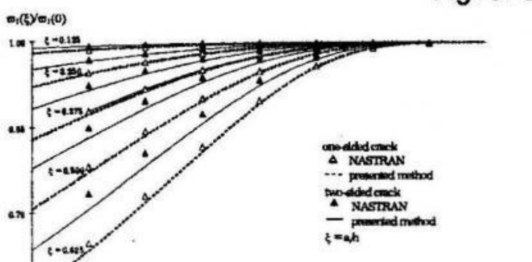

Fig. 4. Influence of crack position and size on the first natural frequency value.

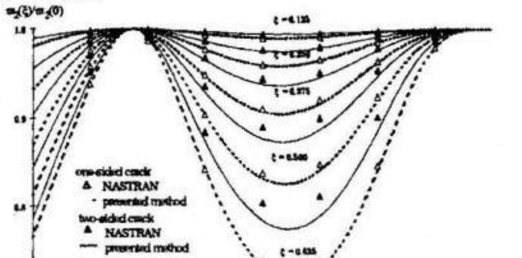

Fig. 5. Influence of crack position and size on the second natural frequency value.

Case 2.

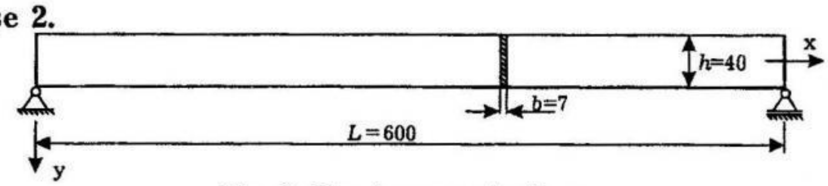

Fig. 6. Simply supported beam.

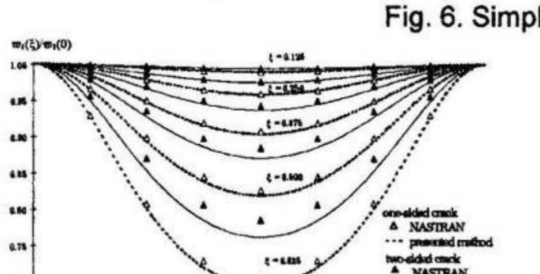

Fig. 7. Influence of crack position and size on the first natural frequency value.

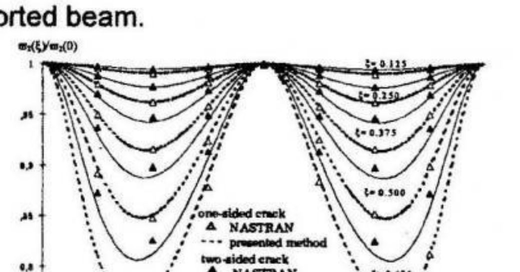

Fig. 8. Influence of crack position and size on the second natural frequency value.

Case 3.

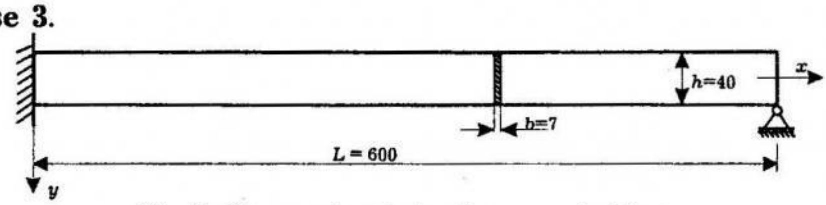

Fig. 9. Clamped and simply supported beam.

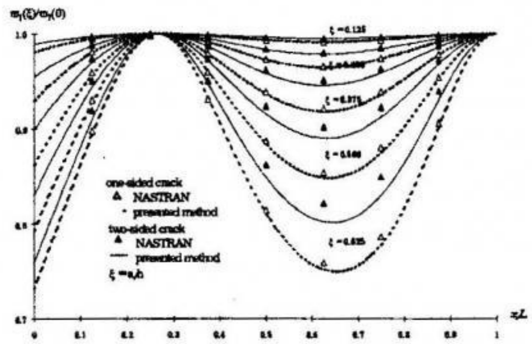

Fig. 10. Influence of crack position and size on the first natural frequency value.

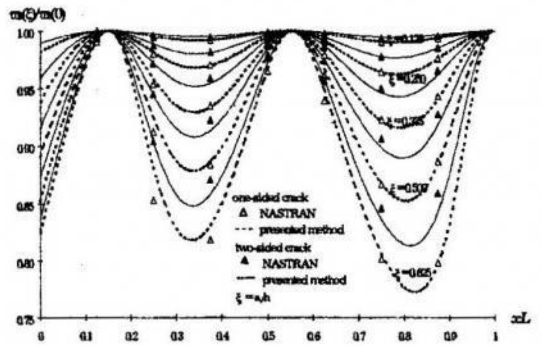

Fig. 11. Influence of crack position and size on the second natural frequency value.

It can be observed in Figs. 4–5, 7–8 and 10–11 that the values of all natural frequencies for the one-sided cracked beams are smaller than those for the two-sided cracked ones. This is because the effective stiffnesses of the latter are larger than that of the former, but total mass is the same in both the systems. The differences between $\varpi_n(\xi)$ and $\varpi_n(0)$ are significant – even up to 25% for $\xi = a/h = 0.625$, with the crack appeared between two nodal points of the modal shapes. The eigenvalues are insensitive to the size of the crack sited at the points coincided with the modal nodes, though.

In the case of statically indeterminate beam (Case 3) an additional nodal point (apart from the conventional nodal points) can be observed. At such a singular point the values of natural frequencies are insensitive to the size of the crack.

The forced vibration problem of the cracked beam described in Case 2 is analysed by using the beam theory, based on the elastic hinge concept. The beam with one- and two-side crack is excited by a concentrated force, $P(t) = P_0 \sin \varpi t$, $\varpi = 175.07\,\text{Hz}$. The crack as well as loading force are defined at the same mid-point of the beam. Obtained nondimensional values of deflection amplitudes y, bending moments M and shear forces T, scaled by corresponding static's quantities $y_s = P_0 L^3/(48EI_{yy})$, $M_s = P_0 L/4$ and $T_s = P_0$, respectively, are shown in Figs. 12, 13 and 14.

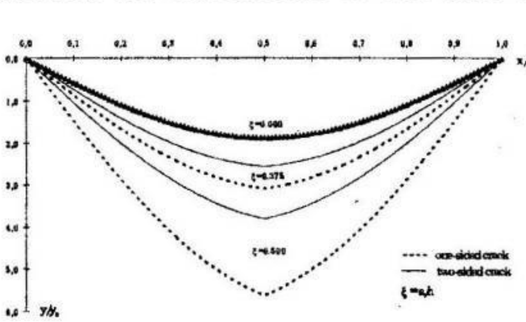

Fig. 12. Beam deflection amplitudes.

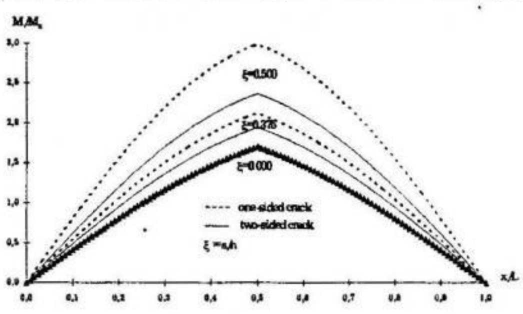

Fig. 13. Bending moment amplitudes.

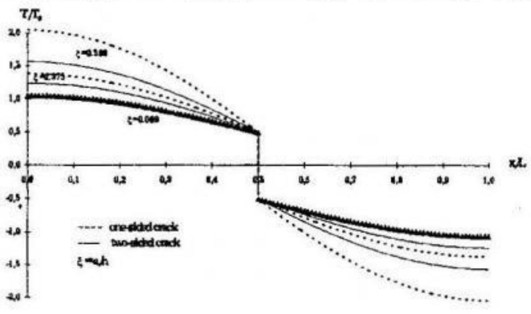

Fig. 14. Shear force amplitudes.

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АНАЛІЗ КОЛИВАНЬ БАЛОК З ТРІЩИНАМИ

Запропоновано метод обчислення частот власних коливань балок з тріщинами, який ґрунтується на класичній теорії Тимошенка, в якій тріщина замінена еквівалентним пружним шарніром з урахуванням впливу обертової інерції та деформації зміни форми. Модель пружного шарніра створена на основі технічної теорії згину балок, порівнюється з моделлю механіки руйнування. Жорсткість шарніра визначена для прямокутного поперечного перерізу з односторонньою та двосторонньою щілиною. Проаналізовано вплив розташування і величини щілини на частоту власних коливань балки. Результати порівняно з обчисленими за допомогою пакета NASTRAN для трьох призматичних балок з прямокутним перерізом. Продемонстровано вплив щілини на амплітуду прогину, на згинальний момент і на поперечну силу для вимушених коливань.

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