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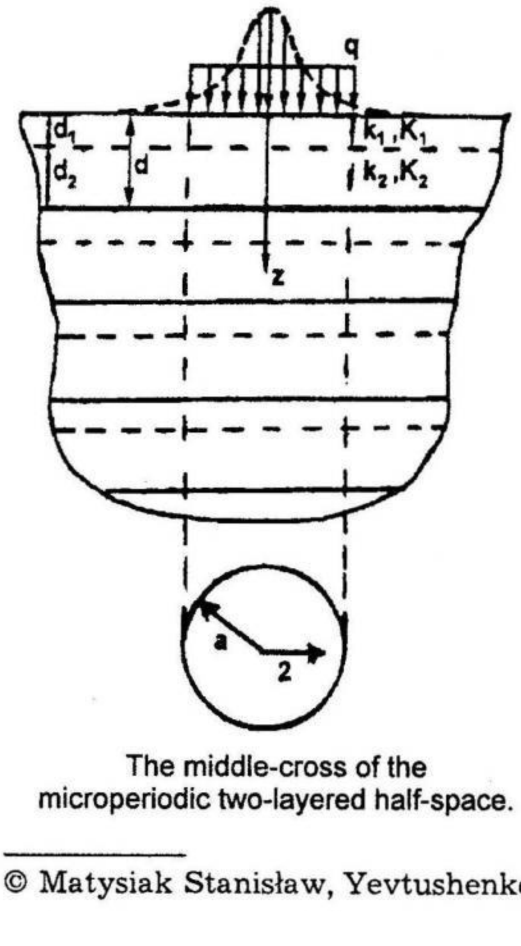
ON NONSTATIONARY THERMOELASTIC PROBLEMS OF PERIODIC LAYERED COMPOSITES

1. Introduction. The problems of temperature and stress distributions in thermoelastic bodies caused by local thermal sources are very important in engineering practice. The non-stationary temperature fields in the homogeneous half-space due to the action of laser beam on the boundary were considered in papers [3–6]. The distributions of temperature and stesses in the homogeneous thermoelastic semi-infinite body caused by a laser ray were investigated in [7–8]. The influence of micro periodically layered structure of composite half-space under the uniform and normal (Gaussian) distributions of heat flux on the boundary on the temperature fields is presented in [9].

This paper is a continuation of the studies [9] concerning the problem of laser heating of thermoelastic microperiodically stratified half-space.



The middle-cross of the microperiodic two-layered half-space.

2. Formulation of the problem. Consider a microperiodic laminated elastic half-space in witch each lamina is composed of two homogeneous isotropic layers. The middlecross section of the body is shown in Figure. Let d_1, d_2 be thicknesses of these layers and $d = d_1 + d_2$ be thickness of the repeated lamina. The composite components are characterized by Lame constants λ_1, μ_1, coefficients of thermal conductivity K_i, coefficients of thermal diffusivity k_i, $i = 1, 2$. The perfect mechanical bouding and ideal thermal contact between the layers are assumed. The problem is related to the cylindrical coordinate system (r, φ, z) such that the z-axis is normal to the layering, see Figure.

The boundary plane of composite half-space is heated by the laser rays described as an uniform and Gausian distributions of heat flux and the plane is assumed to be free of loadings. The problem is solved within the framework of the homogenized model with microlocal parameters [1–2]. The temperature field T(·) and displacement vector **u**(·) are assumed in the form

$$T(r, z, t) = Q(r, z, t) + h(z)\, \varepsilon(r, z, t),$$

$$\mathbf{u}(r, z, t) \cong [u_r, 0, u_2]\,(r, z, t) = [w_r, 0, w_z]\,(r, z, t) + h\,(z)\,[W_r, 0, W_z], \tag{1}$$

where $h : R \to R$ is the known continuous d-periodic function, next defined

$$h(z) = \begin{cases} z - 0.5d_1, & \text{for } 0 \le z \le d_1, \\ -\dfrac{\eta z}{1-\eta} - 0.5d_1 + \dfrac{d_1}{1-\eta}, & \text{for } d_1 \le z \le d, \end{cases} \tag{2}$$

$$h(z) \in 0(d), \quad h'(z) \in 0(1), \quad \forall\, z \in R.$$

where

$$\eta = d_1/d\,. \tag{3}$$

The unknown functions $\theta(\cdot)$, $w_r(\cdot)$, $w_z(\cdot)$, are interpreted as the macrotemperature and macrodisplacements, respectively. The unknown functions $\varepsilon(\cdot)$, $W_r(\cdot)$, $W_z(\cdot)$ are the microlocal thermal and kinematical parameters and they are connected with the microstructure of the body.

The problems of heated periodic stratified half-space are determined by the following initial and boundary conditions :

a) *thermal*

$$\theta(\rho, Z, \tilde{F}_0 = 0), \qquad \text{for } \rho,\ Z \ge 0,$$

$$\frac{\partial\theta}{\partial Z} = -\Lambda\, G_i(\rho), \quad i = 1 \text{ or } 2, \qquad \text{for } \rho > 0,\ \ Z = 0,\ \ \tilde{F}_0 > 0,$$

$$\theta(\infty, Z, \tilde{F}_0) = \theta(\rho, \infty, \tilde{F}_0) = 0, \qquad \text{for } \tilde{F}_0 > 0, \tag{4}$$

where

$$\Lambda = \frac{q_0 a}{K}, \quad \rho = \frac{r}{a}, \quad Z = \frac{z}{a}, \quad \tilde{F}_0 = \frac{k_0\, t}{a^2},$$

$$K_0 = \frac{K}{\tilde{K}}, \quad K = \tilde{K} - \frac{[K]^2}{K}, \quad k_0 = \frac{\tilde{\rho}\tilde{c}}{\tilde{K}},$$

$$\tilde{K} = \eta K_1 + (1-\eta) K_2, \quad \tilde{\rho}\tilde{c} = \eta \rho_1 c_1 + (1-\eta)\rho_2 c_2\,,$$

$$[K] = \eta\,(K_1 - K_2), \qquad K = \eta K_1 + \frac{\eta^2}{1-\eta} K_2\,,$$

$$G_1(\rho) = H(1-\rho), \ \ \rho \ge 0, \qquad G_2(\rho) = \exp(-\rho), \ \ \rho \ge 0, \tag{5}$$

(b) *mechanical*

$$\sigma_{zz}(\rho, Z=0, \tilde{F}_0)=0, \quad \rho \geq 0, \ \tilde{F}_0>0, \quad \sigma_{zr}(\rho, Z=0, \tilde{F}_0)=0, \quad \rho \geq 0, \ \tilde{F}_0>0,$$

$$w_z(\rho, Z, \tilde{F}_0=0)=w_r(\rho, Z, \tilde{F}_0=0)=0, \quad \rho \geq 0, \ Z \geq 0,$$

$$w_z(\infty, Z, \tilde{F}_0)=w_r(\infty, Z, \tilde{F}_0)=w_z(\rho, \infty, \tilde{F}_0)=w_r(\rho, \infty, \tilde{F}_0)=0, \quad \text{for } \tilde{F}_0>0 . \quad (6)$$

The governing equations described the homogenized model are presented in [1].

3. Solution of the problem (a) temperature field

(a) *temperature field*

The temperature θ $(\cdot)$ can be written in the form [9]

$$\theta(\rho, Z, \tilde{F}_0)=\Lambda \int_0^\infty \xi \varphi_i(\xi) \Phi(\xi, Z_0, \tilde{F}_0) J_0(\xi \rho) d\xi, \quad i=1,2, \quad \rho \geq 0, \quad Z \geq 0, \quad \tilde{F}_0 \geq 0, \quad (7)$$

where

$$\Phi(\xi, Z, \tilde{F}_{0)}=\frac{1}{2\xi K_0}\left[\exp(-\xi Z_0) \operatorname{erfc}\left(\frac{Z_0}{2\sqrt{\tilde{F}_0}}-\xi\sqrt{\tilde{F}_0}\right)-\exp(Z_0 \xi) \operatorname{erfc}\left(\frac{Z_0}{2\sqrt{\tilde{F}_0}}+\xi\sqrt{\tilde{F}_0}\right)\right],$$

$$Z_0=\sqrt{K_0}\, Z,$$

where erfc $(\cdot)$ is the complementary error function, $J_0(\cdot)$, $J_1(\cdot)$ are the Bessel functions and

$$\rho_1(\xi)=\frac{1}{\xi} J_1(\xi), \qquad \rho_2(\xi)=\frac{1}{2} \exp(-\xi^2/4). \quad (8)$$

The numerical analysis of the obtained temperature $\theta(\cdot)$ was presented in paper [9]. The following conclusions can be formulated:

1° the temperature on the boundary of composite half-space due to the uniform distribution is higher then due to the Gaussian distribution for the fixed radius of the heating circle

2° the temperature fields are characterized by considerable gradients in the radial and axial directions.

3° the increase of ratio of thermal conductivity $K^*=K_1/K_2$ makes the increase of the temperature. The increase of the ratio of temperature diffusivity $k^*=k_1/k_2$ leads to the reduction of the temperature.

4° assuming that the heated half-space is homogeneous then the temperature given by (7) is reduced to the function of temperature for homogeneous half-space presented in [4].

(b) *mechanical field*

The displacements and stresses in the microperiodic two-layered half-space can be obtained by using three potentials derived for the homogenized model in [10] and Hankel and Fourier integral transforms (with a general trigonometric kernel).The thermoelastic state of stresses can initiate the brittle fracture of material. The knowledge of stresses allows for the optimization of thermal fracturing processes of the layered composite.

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1. *Woźniak Cz.* A nonstandard method of modelling of thermoelastic periodic composites // Int. J. Engng. Sci. – 1987. – **25**. – P. 489–498.
2. *Matysiak S. J., Woźniak Cz.* On the microlocal modelling of thermoelastic composites // J. Techn. Phys. – 1988. – **29**. – P. 85–97.
3. *Hector L. G., Hetnarski R. B.* Thermal stresses in materials due to laser heating, Thermal Stresses, IV, R.B. Hetnarski-editor Elservier Sci. – 1996. – P. 453–531.
4. *Rykalin N. N., Uglov A. A., Zuiev I. V., Kokora A.H.* Laser processing of materials, (in Russian), Machine Construction, Moscow. – 1975.
5. *Rykalin N. N., Uglov A. A., Zuiev I.V., Kokora A.H.*, Laser and electronic beam processing of materials, (in Russian), Machine Construction, Moscow. – 1985.
6. *Duley W. W.* CO_2 laser: Effects and applications. – New York: Acad. Press, 1976.
7. *Lauriello P. J., Chen Y.* Thermal fracturing of hard rock // Trans. ASME, J. of Appl. Mech. – 1973. – No. 4. – P. 909–914.
8. *Yevtushenko A. A., Matysiak S. J., Ivanyk E. G.* On the modelling of laser thermal fracturing of hard rock // Engn. Transactions. – 1997. – **45**. – P. 447–461.
9. *Matysiak S. J., Yevtushenko A. A., Ivanyk E. G.* Temperature field in a microperiodic two-layered composite caused by a circular laser heat source // Heat and Mass Transfer. – 1998. – **34**. – P. 127–133.
10. *Kaczyński A.* Three-dimensional thermoelastic problems of interface cracks in periodic two-layered composites // Engng. Fracture Mech. – 1994. – **48**, No. 6. – P. 783–800.

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ПРО НЕСТАЦІОНАРНІ ТЕРМОПРУЖНІ ЗАДАЧІ ПЕРІОДИЧНО ШАРУВАТИХ КОМПОЗИТІВ

Розглянуто розподіл температури та напружень у періодично шаруватій півплощині, яка піддана дії лазерного нагрівання. Задача розв'язана з використанням гомогенізованої моделі з мікролокальними параметрами [1, 2]. *Розв'язок одержано у вигляді інтегралів Ганкеля, деякі числові результати подано у вигляді графіків.*

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