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DETERMINATION OF TEMPERATURE IN RAIL DURING SLIDING OF WHEEL

1. Statement of the problem. The circular cylinder (the wheel) of radius R is moving with a constant running speed V over the bounding of the semi-space (the rail) and is pressed into it by force P. It is known that in wheel-rail contact the creep caused the sliding within the contact region with the speed V_s. Sliding is accompanied by heat generation in the form of the heat fluxes which are directed in both the wheel and rail. Because of the temperature rise the microstructural changes of subsurface layers in rail is possible. Thus in this paper we consider the problem of the frictional heating of the rail only. For this purpose we now introduce the rectangular coordinate axes Oxy, rigidly connected to the leading edge of the wheel. To expedite calculation, the following assumptions were made:

- the width of the contact area $2a$ is small with comparison of the wheel radius R;
- the creep due to elastic deformation is neglected here. For gross sliding (the creep $0.1\% < s < 2\%$) the sliding speed V_s is equal $V_s = s\,V$;
- at the leading edge of the contact zone, only a small temperature difference between wheel and rail can be observation. Thus, the heat partitioning factor becomes $\lambda = 0.5$; the intensity of the heat flux q on the contact area $0 \le x \le a$ is equal $q = \lambda f V_s\, p$, where f is the coefficient of friction, p is the contact pressure;
- the convective heat transfer outside of the contact region is neglected;
- the fast-moving of the frictional heat sources is considered;
- the steady state in the rail is taking into consideration;
- the mechanical and thermal properties of the rail material is constant.

Under these assumptions the heat conductivity problem can be written as

$$\frac{\partial^2 T}{\partial \eta^2} = \frac{\partial T}{\partial \xi}, \quad |\xi| < \infty, \quad 0 < \eta < \infty. \tag{1}$$

$$\left.\frac{\partial T}{\partial \eta}\right|_{\eta=0} = \begin{cases} -\Lambda p^*(\xi), & 0 \le \xi \le 1 \\ 0, & -\infty < \xi < 0,\ 1 < \xi < \infty \end{cases}, \tag{2}$$

$$T \to 0 \quad \text{at} \quad \sqrt{\xi^2 + \eta^2} \to \infty, \tag{3}$$

where

$$\xi = \frac{x}{2a},\ \eta = \frac{y}{d},\ p^* = \frac{p}{p_0},\ \ p_0 = \frac{P}{2a},\ \ d = \sqrt{\frac{2ak}{V_s}},\ \ \Lambda = \frac{fV_s p_0 d}{K},$$

T is the temperature, K is the conductivity, k is the diffusivity.

2. Method of solution. The solution of the heat conductivity problem (1)–(3) is obtained by applying the Fourier transform in the form [6]

$$T(\xi,\eta) = \frac{\Lambda}{\sqrt{\pi}} \int_0^b G(\xi - \tau, \eta)\, p^*(\tau)\, d\tau, \tag{4}$$

$$G(\xi,\eta) = \frac{\exp\left(-\eta^2/(4\xi)\right)}{\sqrt{\xi}}, \quad b = \begin{cases} 0, & -\infty < \xi < 0 \\ \xi, & 0 \le \xi \le 1 \\ 1, & 1 < \xi < \infty \end{cases}.$$

Since $p^*(\tau)$ is arbitrary, Ling and Yang [6] have proposed a Fourier series representation for the contact pressure. In present paper for this purpose we use the piecewice-linear functions (roof functions) method [2]. We shall introduce a uniform mesh into the integration interval $[0, b]$: $0 = \tau_0 < \tau_1 < \ldots < \tau_{n-1} < \tau_n = b$, $\tau_i = i\delta\tau$, $\delta\tau = b/n$, $i = 0, 1, \ldots, n$. Corresponding with every knot τ_i of the mesh a «roof» functions will be put:

$$\varphi_0(\tau) = \begin{cases} (\tau_1 - \tau)/\delta\tau, & \tau \in [\tau_0, \tau_1] \\ 0, & \tau \notin [\tau_0, \tau_1] \end{cases}, \quad \varphi_n(\tau) = \begin{cases} (\tau - \tau_{n-1})/\delta\tau, & \tau \in [\tau_{n-1}, \tau_n] \\ 0, & \tau \notin [\tau_{n-1}, \tau_n] \end{cases},$$

$$\varphi_i(\tau) = \begin{cases} (\tau - \tau_{i-1})/\delta\tau, & \tau \in [\tau_{i-1}, \tau_i] \\ (\tau_{i+1} - \tau)/\delta\tau, & \tau \in [\tau_i, \tau_{i+1}] \\ 0, & \tau \notin [\tau_{i-1}, \tau_{i+1}] \end{cases}, \quad i = 1, 2, \ldots, n-1.$$

We shall build approximation of function $p^*(\tau)$ in the form

$$p^*(\tau) = \sum_{i=0}^{n} p_i^* \varphi_i(\tau), \quad p_i^* \equiv p^*(\tau_i). \tag{5}$$

The uniform error of this approximation for $p^*(\tau) \in C^2([0,b])$ has order $O(\delta\tau^2)$ [2].

Substituting the approximation (5) into the equation (4) we find

$$T(\xi,\eta) = \begin{cases} 0, & -\infty < \xi < 0 \\ \dfrac{\Lambda}{\sqrt{\pi}\,\delta\tau} \displaystyle\sum_{i=0}^{n} p_i^* L_i(\xi,\eta), & 0 \le \xi \le 1 \\ \dfrac{\Lambda}{\sqrt{\pi}\,\delta\tau} \displaystyle\sum_{i=0}^{n} p_i^* M_i(\xi,\eta), & 1 < \xi < \infty \end{cases}, \tag{6}$$

where

$$M_0(\xi,\eta) = \tau_1 J_0(\tau_1,\xi,\eta) - J_1(\tau_1,\xi,\eta)$$

$$M_n(\xi,\eta) = J_1(\tau_n,\xi,\eta) - J_1(\tau_{n-1},\xi,\eta) - \tau_{n-1}[J_0(\tau_n,\xi,\eta) - J_0(\tau_{n-1},\xi,\eta)]$$

$$M_i(\xi,\eta) = \tau_{i-1}J_0(\tau_{i-1},\xi,\eta) - (\tau_{i+1} + \tau_{i-1})J_0(\tau_i,\xi,\eta) + \tau_{i+1}J_0(\tau_{i+1},\xi,\eta) - $$
$$- J_1(\tau_{i-1},\xi,\eta) + 2J_1(\tau_i,\xi,\eta) - J_1(\tau_{i+1},\xi,\eta), \quad i = 1, 2, ..., n-1;$$

$$J_0(\tau_i,\xi,\eta) = -2\sqrt{\xi-\tau_i}\exp\left(-\frac{\eta^2}{4(\xi-\tau)}\right) + 2\sqrt{\xi}\exp\left(-\frac{\eta^2}{4\xi}\right) -$$
$$-\eta\sqrt{\pi}\left[\operatorname{erf}\left(\frac{\eta}{2\sqrt{\xi-\tau_i}}\right) - \operatorname{erf}\left(\frac{\eta}{2\sqrt{\xi}}\right)\right], \quad \xi > \tau_i, \quad i = 1,2,..., n;$$

$$J_1(\tau_i,\xi,\eta) = \frac{2}{3}(\xi-\tau_i)\sqrt{\xi-\tau_i}\exp\left(-\frac{\eta^2}{4(\xi-\tau_i)}\right) - \frac{2}{3}\xi\sqrt{\xi}\exp\left(-\frac{\eta^2}{4\xi}\right) +$$
$$+\left(\frac{\eta^2}{6}+\xi\right)J_0(\tau_i,\xi,\eta), \quad \xi > \tau_i, \quad i = 1, 2, ..., n. \tag{7}$$

The functions $L_i(\xi, \eta)$ in the equations (6) were calculated using the formulae (7) at substitution of functions $J_k(\tau_i, \tau_i, \eta)$, $i = 1, 2, ..., n$; $k = 0, 1$ instead of functions $J_k(\tau_i, \xi, \eta)$, where

$$J_0(\tau_i,\tau_i,\eta) = -2\sqrt{\tau_i}\exp\left(-\frac{\eta^2}{4\tau_i}\right) - \eta\sqrt{\pi}\operatorname{erfc}\left(\frac{\eta}{2\sqrt{\tau_i}}\right),$$

$$J_1(\tau_i,\tau_i,\eta) = -\frac{2}{3}\tau_i\sqrt{\tau_i}\exp\left(-\frac{\eta^2}{4\tau_i}\right) + \left(\frac{\eta^2}{6}+\tau_i\right)J_0(\tau_i,\tau_i,\eta), \quad i = 1,2,..., n.$$

3. Contact pressure. The distribution of the contact pressure has the form [4]

$$p^*(\tau) = \frac{\sin(\pi\alpha)}{\pi\alpha\beta}\tau^\alpha(1-\tau)^\beta, \tag{8}$$

where

$$\alpha = \frac{1}{\pi}\operatorname{arctg}\left(\frac{1}{f|B-H|}\right), \quad \beta = 1-\alpha, \quad 0 < \alpha,\beta < 1,$$
$$B = \frac{1-2\nu}{2(1-\nu)}, \quad H = \frac{2\delta k\mu}{1-\nu}, \quad \delta = \frac{\alpha_t(1+\nu)}{K}. \tag{9}$$

The values of the parameters B and H for a large range of materials are given in [4]. Note that the parameter H is remarkably close to unity for most metals.

At $H = 0$ from the formulae (8) and (9) we obtain the known solution of the isothermal contact problem [1]. If $f = 0$ or $H = B$ then $\alpha = \beta = 1/2$ and we have the solution of the plane Hertz contact problem [1].

4. Numerical example and conclusions. The numerical examples has been considered for the friction couple wheel-rail at $R = 0.5$ m; $f = 0.3$; $\nu = 0.3$; $\mu = 80.8$ GPa; $K = 41$ W/(m·K); $k = 9.1\cdot10^{-6}$ m^2/s; $\alpha_t = 10^{-5}$ K^{-1} because of the experimental data [5] for three cases of the load. These results are listed in the following table

Parameters	Passenger carstatic load	Locomotive wheelstatic load	Locomotive wheeldynamic load
P (N/m)	10^5	$2{\cdot}10^5$	$2{\cdot}10^5$
V (m/s)	25	75	75
s,%	0,1	1	2
V_s (m/s)	0,025	0,75	1,5
a (mm)	0,37	0,52	0,74
p_0 (MPa)	66,7	94,3	133,4
T_{max} (K)	1,5	13,8	32,9

It is observed that the maximum contact temperature is reached in the case at high running speed (V = 75 m/s) with a creep of 2% for the dynamic locomotive wheel load. The calculated temperatures do not exceed 350°C. Thus, the thermally induced phase transformation of the rail material, where the temperature of at least 600°C would be needed [6], cannot be induced.

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ВИЗНАЧЕННЯ ТЕМПЕРАТУРИ У РЕЙЦІ ПІД ЧАС КОВЗАННЯ КОЛЕСА

Запропоновано математичну модель визначення температурного поля під час тертя внаслідок ковзання залізничного колеса по рейці. Використано відомий розв'язок Лінга плоскої квазістаціонарної задачі теплопровідності. На базі методу кусково-лінійної апроксимації побудовано розрахункову схему та виконано числовий аналіз для декількох типів інтенсивності фрикційного теплового потоку.

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