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CONTACT PROBLEM WITH THE FRICTIONAL HEATING FOR A HALF-SPACE AND A PUNCH WITH BOUNDARY RECTANGULAR-SHAPE WAVINESS

1. Introduction. The contact problem for an elastic half-space and the rigid punch having the profile of periodically distributed rectangular waviness was considered in [4]. Recently [3] we studied the contact with frictional heating of bodies with the boundary parabolic waviness. In this work we consider the following contact problem. The rigid thermoinsulated punch is pressed to the thermoelastic half-space $y < 0$ and slides with the constant velocity V in z-axis direction (Fig. 1). It is assumed that the punch has the profile of rectangular waviness with the period L and the contact occurs on the region $\Gamma = \bigcup_{k=-\infty}^{k=+\infty} (a_k, b_k)$. The friction forces σ_{zy} which have place in the contact area are described by Amonton's law. Moreover, these forces produce the heat which conducts into the half-space only. Outside the contact region the Newton's type of heat exchange is allowed. The problem is considered as planar.

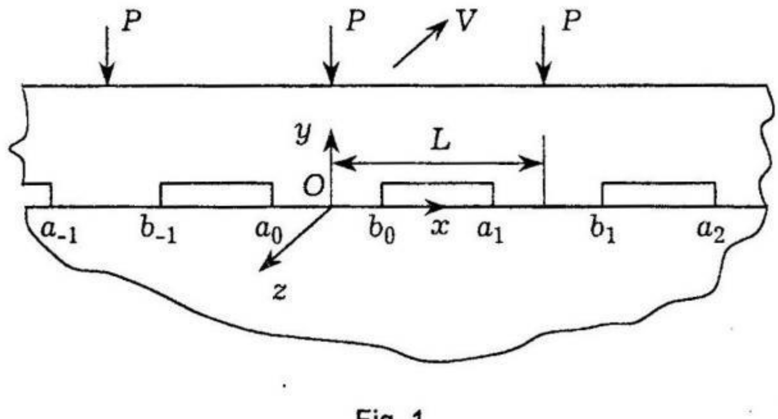

Fig. 1.

2. System of integral equations of the problem. Main idea in the construction of integral equations of the problem under consideration is that the contact pressure, heat flux, temperature are the very same in all contact regions (a_k, b_k), $k = -\infty, \dots, +\infty$, or to be more exact, all fields in the half-space are L-periodic functions. Under this assumption it is possible to write the integral equations on the simple contact region (a_0, b_0). This approach, proposed in [4], was applied in [3] to the boundary problems of thermoelasticity. The system of integral equations of problem formulated above has the form [3]

$$\frac{1-\nu}{\pi\mu}\int_{a_0}^{b_0}\frac{p(x')}{x-x'}dx' - \frac{1-\nu}{\mu L}\int_{a_0}^{b_0}p(x')G(x-x')\,dx' - \frac{fV\delta}{\pi}\int_{a_0}^{b_0}p(x')N(x-x')\,dx' +$$

$$+\frac{\delta h}{\pi}\int_{a_0}^{b_0}T(x')N(x-x')\,dx' = g'(x), \qquad a_0 \le x \le b_0, \tag{1}$$

$$T(x) - \frac{h}{\pi K}\int_{a_0}^{b_0}T(x')M(x-x')\,dx' + \frac{fV}{\pi K}\int_{a_0}^{b_0}p(x')M(x-x')\,dx' = 0, \quad a_0 \le x \le b_0, \tag{2}$$

$$\int_{a_0}^{b_0}p(x')\,dx' = P, \tag{3}$$

where ν, μ, K, δ, h are, respectively, Poisson's ratio, shear module, thermal conductivity, thermal distortion and radiation coefficient of the half-space; $p(x)$ is contact pressure, $T(x)$ is temperature of the half-space boundary; f is friction coefficient; P is load applied within one period; $g(x)$ is the function describing the waviness geometry, which is equal $g(x) = \text{const}$ for rectangular-shape waviness. Kernels of these integral equations have the forms

$$G(z) = \frac{1}{3}\left(\frac{\pi z}{L}\right) + \frac{1}{45}\left(\frac{\pi z}{L}\right)^3 + \frac{2}{945}\left(\frac{\pi z}{L}\right)^5 + \frac{1}{4725}\left(\frac{\pi z}{L}\right)^7 + \dots,$$

$$N(z) = 2\pi\sum_{j=1}^{\infty}\frac{\sin(2\pi jz/L)}{2\pi j + hL/K}, \qquad M(z) = \frac{\pi K}{hL} + 2\pi\sum_{j=1}^{\infty}\frac{\cos(2\pi jz/L)}{2\pi j + hL/K}.$$

Introducing dimensionless variables and functions

$$r = x'/a, \qquad s = x/a, \qquad p^*(r) = \frac{a}{P}p(x), \qquad T^*(r) = \frac{K}{fVP}T(x),$$

$$\beta = \frac{fV\delta\mu a}{1-\nu}, \qquad \lambda = L/a, \qquad Bi = \frac{ha}{K}, \qquad a = \frac{1}{2}(a_0 + b_0) \tag{4}$$

the system (1)–(3) can be rewritten in the dimensionless form

$$\frac{1}{\pi}\int_{-1}^{1}p^*(r)\left\{\frac{1}{s-r} - \frac{\pi}{\lambda}G^*(s-r) - \beta N^*(s-r)\right\}dr +$$

$$+\frac{\beta\,\mathrm{Bi}}{\pi}\int_{-1}^{1}T^*(r)N^*(s-r)\,dr = 0, \quad |s| \le 1, \tag{5}$$

$$T^*(s) - \frac{Bi}{\pi}\int_{-1}^{1}T^*(r)M^*(s-r)\,dr + \frac{1}{\pi}\int_{-1}^{1}p^*(r)M^*(s-r)\,dr = 0, \quad |s| \le 1, \tag{6}$$

$$\int_{-1}^{1} p^*(r)\,dr = 1, \tag{7}$$

where kernels $G^*(\cdot)$, $M^*(\cdot)$, $N^*(\cdot)$ are some dimensionless functions.

3. Numerical solution of the system of integral equations. The integral equation (5) is of the Cauchy-type with the index equal to 1. In this case the distribution of contact pressure can be written as

$$p^*(s) = \varphi(s)/\sqrt{1-s^2}, \tag{8}$$

where $\varphi(s)$ is a new unknown function. The integral equation (6) is of the Fredholm-type of second kind with the weakly-singular kernel. So the temperature will be consider of the class of limited functions.

Using the Gauss-Chebyshev quadrature formulae [2] and rectangular quadrature, the system (5)–(7) can be transformed to the system of linear algebraic equations

$$\frac{1}{\pi}\sum_{k=1}^{n}\varphi(r_k)\,w_k\left\{\frac{1}{s_m-r_k}-\frac{\pi}{\lambda}G^*(s_m-r_k)-\beta N^*(s_m-r_k)\right\}+$$

$$+\frac{2\beta\,\mathrm{Bi}}{\pi n}\sum_{k=1}^{n}T^*(\rho_k)N^*(s_m-\rho_k)=0, \qquad m=1,\ldots,n-1,$$

$$T^*(\rho_m)-\frac{Bi}{\pi}\sum_{k=1}^{n}T^*(\rho_k)\,A_{km}+\frac{1}{\pi}\sum_{k=1}^{n}\varphi^*(r)\,w_k\,M^*(\rho_m-r_k)=0, \quad m=1,\ldots,n,$$

$$\sum_{k=1}^{n}\varphi^*(r)\,w_k=1, \tag{9}$$

where $r_k=\cos\dfrac{2k-1}{2n}\pi, \qquad w_k=\dfrac{\pi}{n}, \qquad k=1,\ldots,n;$

$$s_m=\cos\frac{m}{n}\pi, \quad m=1,\ldots,n-1; \quad \rho_k=-1+\frac{1}{n}(2k-1), \quad k=1,\ldots,n;$$

$$A_{km}=\frac{2\pi}{n\lambda\,\mathrm{Bi}}+\sum_{j=1}^{\infty}\frac{1}{j(2\pi j+\lambda\,\mathrm{Bi})}\left\{\sin\frac{2\pi jX_1}{\lambda}-\sin\frac{2\pi jX_2}{\lambda}\right\}, \qquad k,m=1,\ldots,n;$$

$$X_1=(2m-2k+1)/n, \qquad X_2=(2m-2k-1)/n, \qquad k,m=1,\ldots,n.$$

The system of $2n$ equations (9) is closed for $2n$ unknowns $\varphi(r_k)$, $T^*(\rho_k)$, $k=1,\ldots,n$.

4. Results. There are following input parameters of the problem: Biot's coefficient Bi, dimensionless period λ and parameter β which can be considered as a frictional heating ratio. The numerical calculations showed that the Biot's coefficient has not effects on the contact pressure distribution but has the great influence on the contact temperature. Analyzing the effects of the parameters λ and β on the contact pressure, some interesting results were obtained. For some values of these parameters the function $\varphi(s)$ changes the sign for $s=\pm1$. This can be treated as a perturbation of the contact at corner points of the punch. The values of λ and β for which the contact in points $s=\pm1$ is lost are placed above the solid line in Fig. 2. Below this line the contact in the region $|s|\le 1$ is full. For $\lambda\to\infty$ the well-known result β_{cr}=1.16 [1] for critical value of the frictional heating ratio in the single punch problem was approached.

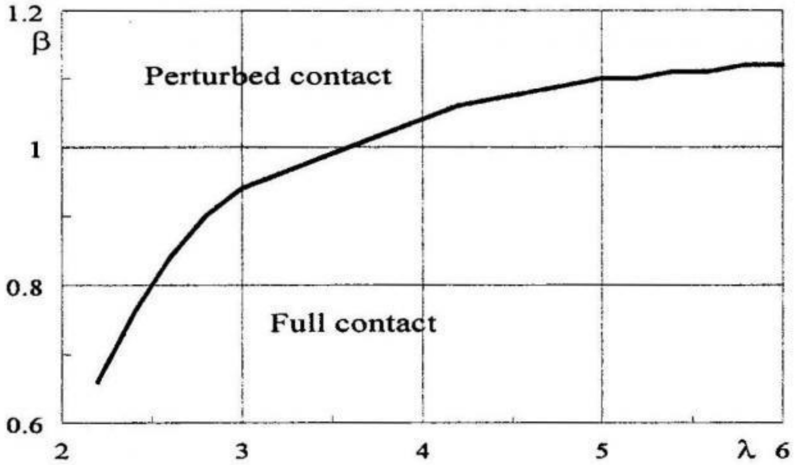

Fig. 2.

It is clear that the perturbations of the contact have place for small values of the period λ (i.e. for a «dense» waviness) and for great values of the ratio β (i.e. for high speed V, great friction coefficient f, etc., See (4)).

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КОНТАКТНА ЗАДАЧА З ТЕПЛОУТВОРЕННЯМ ДЛЯ ПІВПРОСТОРУ З ПРЯМОКУТНО ХВИЛЯСТОЮ ПОВЕРХНЕЮ

Досліджено контактну взаємодію пружного теплопровідного півпростору та жорсткого теплоізольованого рухомого штампа. Вважається, що основа штампа має форму прямокутної періодичної хвилястості. Взято до уваги явище виділення тепла внаслідок тертя тіл. Для розв'язування задачі використано метод сингулярних інтегральних рівнянь. Досліджено вплив поверхневої структури штампа на розв'язок задачі.

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