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ON ASSOCIATED GROUPS OF RINGS SATISFYING FINITENESS CONDITIONS

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We consider the construction of associated group of a ring with identity element. The characterization of rings with periodic, FC-group, nilpotent associated group are given. It is shown that some finiteness conditions or commutativity of a ring R follow from the finiteness conditions of the associated group $G(R)$.

Key words: associated group of a ring, adjoint group, FC-group, periodic group.

1. Let R be an associative ring with an identity element. The set of all elements of R forms a semigroup with the identity element $0 \in R$ under the operation $a \circ b = a + b + ab$ for all a and b of R. The group of all invertible elements of this semigroup is called the *adjoint group* of R and is denoted by R°. Clearly, if R has the identity 1, then $1 + R^\circ$ coincides with the group of units $U(R)$ of the ring R and the map $a \to 1 + a$ with $a \in R$ is an isomorphism from R° onto $U(R)$.

Many authors have studied the rings with prescribed adjoint groups (or equivalently, groups of units) (see, for example, [1-16]).

This paper is concerned with the question of how properties of associated group influence some characteristic of rings structure. The idea of associated group was introduced in [1] for radical ring. We extend this construction to arbitary associative rings with identity element.

In Sections 3,4,5 we obtain some results on rings determined by their associated groups which are periodic, FC-groups, nilpotent groups. It is proved that finiteness conditions of the associated group $G(R)$ imply some finiteness conditions or commutativity of a ring R.

2. Preliminaries. Let R be an associative ring (not necessarily with identity element) and R° its adjoint group. In the same way as in [1] we consider the set of pair $G(R) = \{(x, y) \mid x \in R, y \in R^\circ\}$ and define an operation by the rule

$$(x, y)(u, v) = (y \cdot u + u + x, y \circ v). \tag{2.1}$$

Definition 2.1. *Let R be an associative ring. Then $G(R) = A \rtimes B$ is a group with the neutral element $(0,0)$ with respect to the operation (2.1), where $A = \{(x, 0) \mid x \in R\} \cong R^+$, $B = \{(0, y) \mid y \in R^\circ\} \cong R^\circ$.*

Following [1], the group $G(R)$ will be called the *associated group* of the ring R.

Lemma 2.2. *Let R be an associative ring with associated group $G(R)$. If S is a subring of R with associated group $G(S) = X \rtimes Y$ then following statements are true:*

(i) $G(S) \leqslant G(R)$, $X \leqslant A$, $Y \leqslant B$;

(ii) if S is a left ideal of the ring R, then $X \lhd G(R)$;

(iii) if $X \lhd G(R)$, then $rS \leqslant S$ for all $r \in R$;

(iv) if S is a right ideal of the ring R, then $G(S) \lhd A \rtimes Y$;

(v) if $G(S) \lhd A \rtimes Y$, then $S^{\circ} R \leqslant S$;

(vi) if S a two-side ideal of the ring R, then $G(S) \lhd G(R)$, $X \lhd G(R)$;

(vii) $C_A(B) = \{(a,0) \mid a \in Ann_r(R^{\circ})\}$, $C_B(A) = \{(0,b) \mid b \in R^{\circ}$ and $b \in Ann_l(R)\}$; in particular, if R is a ring with identity, then $C_B(A) = \langle(0,0)\rangle$ and if R is a domain, then $C_B(A) = C_A(B) = \langle(0,0)\rangle$.

Proof. (i) is immediate from Definition 2.1.

(ii) Let S be a left ideal of ring R and $rs \in S$ for all elements $r \in R$ and for all elements $s \in S$. Then for an arbitrary element $(a,b) \in G(R)$ and arbitrary element $(x,0) \in X$ we have

$$(a,b)^{-1}(x,0)(a.b) = (b^{(-1)}x + x, 0) \in X, \tag{2.2}$$

hence X is a normal subgroup in $G(R)$.

(iii) If $X \lhd G(R)$, then (2.2) implies that $b^{(-1)}x \in S$ for all $b \in R^{\circ}$ and all $x \in S$.

(iv) Let S be a right ideal of the ring R and $sr \in S$ for all $s \in S$, $r \in R$. Then for all elements $(x,y) \in X \rtimes Y$ and all $(a,c) \in A \rtimes Y$ we have

$$\begin{gathered}(a,c)^{-1}(x,y)(a,c) = (-c^{(-1)}a - a, c^{(-1)})(x,y)(a,c) = \\ (ya + c^{(-1)}ya + c^{(-1)}x + x,\ y + c^{(-1)}y + yc + c^{(-1)}yc) \in X \rtimes Y,\end{gathered} \tag{2.3}$$

because $c, y \in S$. Therefore $G(R) \lhd A \rtimes Y$.

(v) If $c = 0$, then (2.3) yields $S^{\circ} R \leqslant S$.

(vi) Since S is a two-side ideal of the ring R, for arbitrary elements $(x,y) \in X \rtimes Y$ and $(u,v) \in G(R)$ we have

$$\begin{gathered}(u,v)^{-1}(x,y)(u,v) = \\ (yu + v^{(-1)}yu + v^{(-1)}x + x,\ y + v^{(-1)}y + yv + v^{(-1)}yv) \in G(S).\end{gathered} \tag{2.4}$$

In particular, if $y = 0$ then $(u,v)^{-1}(x,0)(u,v) = (v^{(-1)}x + x, 0) \in X$, hence $X \lhd G(R)$.

(vii) Let $(a,0) \in C_A(B)$. Then for arbitrary elements $(0,b) \in B$ we have

$$(0,b) = (a,0)^{-1}(0,b)(a,0) = (ba, b) \tag{2.5}$$

and consequently $ba = 0$ for all $b \in R^{\circ}$. Therefore $a \in Ann_r(R^{\circ})$. The converse statement is also true.

Let $(0,b) \in C_B(A)$. Then for all elements $(a,0) \in A$ we have

$$(a,0) = (0,b)^{-1}(a,0)(0,b) = (b^{(-1)}a + a, 0) \tag{2.6}$$

and hence $b^{(-1)}a = 0$ for all $a \in R$. It follows that

$$0 = 0 \cdot a = (b + b^{(-1)} + bb^{(-1)})a = ba, \tag{2.7}$$

hence $b \in Ann_l(R)$.

Lemma 2.3. *Let R be a ring and I be an ideal of R such that $I \leqslant J(R)$. Then*

$$G(R/I) \cong G(R)/G(I). \tag{2.8}$$

Proof. Let $G(R) = A \rtimes B$ (respectively $G(I) = X \rtimes Y$, $G(R/I) = C \rtimes D$) be an associated group of the ring R (respectively of the ideal I, of the quotient-ring R/I). Then

$$\begin{gathered} G(R)/G(I) = AB/G(I) \cong AG(I)/G(I) \cdot BG(I)/G(I) = \\ (AXY/XY) \rtimes (BXY/XY) = (AY/XY) \rtimes (XB/XY). \end{gathered} \tag{2.9}$$

Moreover,

$$\begin{gathered} D \cong (R/I)^{\circ} \cong R^{\circ}/I^{\circ} \cong B/Y \cong XB/XY, \\ C \cong (R/I)^{+} \cong R^{+}/I^{+} \cong A/X \cong AY/XY. \end{gathered} \tag{2.10}$$

(2.8) is immediate from the above equations.

The next corollary follows from Lemma 4.2 [3].

Corollary 2.4. *Let S be unital subring of ring R such that $|R^{+} : S^{+}| < \infty$. Then $|G(R) : G(S)| < \infty$.*

3. Rings with Periodic Associated Group. By analogy with Lemma 1.1 [3] the following lemma can be proved.

Lemma 3.1. *Let R be a ring and $J = J(R)$ its Jacobson radical. Then $G(R)$ is a periodic group if and only if J is a nil ideal with periodic additive group J^{+} and the group $G(R/J)$ is periodic.*

Remark 3.2. *It is clear that for any ring R with identity the following statements are equivalent:*

1) the group $G(R)$ is periodic if and only if so is the group of units $U(R)$;

2) $charR$ is finite.

Let us recall that a field T is *absolute* if T is a field of prime characteristic p and T is an algebraic extension of its simple subfields. Hence the multiplicative group T^{*} of an absolute field T is a periodic p'-group.

Lemma 3.3. *Let R be a comutative ring with identity. Suppose that R has no zero divisors and $Q(R)$ its field of quotients. Then $G(Q(R))$ is a periodic group if and only if R is an absolute field.*

Proof. ($\Leftarrow$) Sufficiency of the lemma is clear.

($\Rightarrow$) Suppose that $G(Q(R))$ is a periodic group. Then for all elements $r \in R$ there exists $n = n(r) \in \mathbb{N}$ such that $r^{n} = 1$. Therefore the element r is invertible in R. The lemma is proved.

Theorem 3.4. *Let R be a ring with identity and suppose that R has no zero divisors. Then $G(R)$ is periodic group if and only if the following statements are equivalent:*

1) $P[x]$ is a field, where P is simple subfield of R;

2) the element $x \in R$ is algebraic over P;

3) $x \in U(R)$.

Proof. Necessity. Suppose that the group $G(R)$ is periodic. Then $charR = p$, where p is prime.

(1) $\Rightarrow$ (2). If $P[x]$ is a field, then the element x is invertible. It follows that $x^n = 1$ for some $n \in \mathbb{N}$, hence x is algebraic over P.

(2) $\Rightarrow$ (1). If x is algebraic over P, then the domain $P[x]$ is finite and therefore it is a field.

Implications (3) $\Rightarrow$ (2) and (1) $\Rightarrow$ (3) are obvious.

Sufficiency. Suppose that the items (1), (2) and (3) are equivalent for the ring R. Assume the contrary, that a is an element of infinite order in the adjoint group R°. Then $1 + a \in U(R)$, hence $P[1 + a]$ is a field and the condition (2) imply that element a is algebraic over P. This contradiction completes the proof.

Corallary 3.5. *Let R be a ring with identity, P be a prime subring of R. If R has no zero divisors, then $R^\circ = \{0\}$ if and only if the following statements are true:*

1) $P \cong GF(2)$;

2) any element $x \in R - P$ is transcendental over P;

3) $P[x]$ is not a field for arbitrary element $x \in R - P$.

Proof. Suppose $R^\circ = \{0\}$, then $2 = -2$ and therefore $char R = 2$. Assume that there exists an element $a \in R - P$ algebraic over P. Then $P[a]$ is a finite ring without zero divisors. It means that $P[a]$ is a field and $a \in U(R)$, giving a contradiction. So condition (2) is true. Condition (3) is obvious. The converse is trivial.

The rings R with torsion free additive group R^+ and periodic group of units $U(R)$ were studied in paper [5].

Remark 3.6. *If $K[G]$ is a group ring, of a non-trivial group G over a skew field K of zero characteristic, then the group of units $U(K[G])$ is not periodic.*

Indeed, if $char K = 0$, then the prime subfield P of skew field K is isomorphic to $\mathbb{Q}$, but $\mathbb{Q}^*$ is not a periodic group.

Corollary 3.7. *Let $K[H]$ be a group algebra of a group H over a skew field K. Then the following statements are equivalent:*

1) $G(K[H])$ is a periodic group;

2) $U(K[H])$ is a periodic group;

3) K is an absolute field, H is a locally finite group.

Proof. (1) $\Leftrightarrow$ (2) is obvious.

(2) $\Rightarrow$ (3). Since the groups H and K^* can be embedded in $U(K[H])$, it follows from Lemma 2.1 [15] that K is an absolute field and H is a periodic group.

Let $y_1, \dots, y_n$ be arbitrary elements of the group H. Since $K = \bigcup_{i=1}^{\infty} K_i$, where K_i are finite fields and $K_i[y_1, \dots, y_n]$ are finite domains (hence fields), the subgroup $\langle y_1, \dots, y_n \rangle \leqslant H$ is finite.

(3) $\Rightarrow$ (2). Clearly, for any element $x \in K[H]$ there exists a finite subfield F of the field K such that $x \in F[C]$ for certain finite subgroup C of the group H. Since subring $F[C]$ is finite, the group $U(K[H])$ is periodic.

4. Associated Groups with Finite Conjugacy Classes. A group G is called an *FC-group* if every conjugacy class is finite, i.e., if $|G : C_G(x)| < \infty$ for all element $x \in G$.

Lemma 4.1. *Let R be a ring with identity. Then $G(R)$ is an FC-group if and only if $G(R)$ is a locally normal group.*

Proof. Let $G(R) = A \rtimes B$, where $A \cong R^+$ and $B \cong R^\circ$. If the group R° is not periodic, then by Corollary 3.10 [20] $C_B(A) \neq 1$. But it contradicts Lemma 2.2 (vii). Therefore, the subgroup R° is periodic. Let $(a, 0)$ be an arbitrary element of A. Since $(a, 0)^n \in Z(G(R))$ for some $n = n(a) \in \mathbb{N}$, we obtain

$$(na, b) = (na, 0)(0, b) = (0, b)(na, 0) = (bna + na, b). \tag{4.1}$$

Hence,

$$bna = 0 \tag{4.2}$$

for arbitrary non-zero element $a \in R$.

If $char R = 0$, then $(-2)e \in R^\circ$, where e is the identity element of the ring R. From (4.2), if we put $a = e$ we get $nb = 0$ for arbitrary $b \in R^\circ$. It contradicts that the order $|-2e|_+$ is infinite. Therefore $char R = n$ is finite. Thus $G(R)$ is a locally normal group. The converse is trivial. The lemma is proved.

Corollary 4.2. *Let R be a ring with identity. Then $G(R)$ is a fibrewise finite group if and only if R is a finite ring.*

Corollary 4.3. *Let R be a ring with identity. Suppose R has no zero divisors, then $G = G(R)$ is an FC-group if and only if $R^\circ = \{0\}$ or R is a finite field.*

Indeed, if the adjoint group R° is not trivial, then it follows from Lemma 4.1 and fact, that quotient-group $G/C_G(x^G)$ (where $x^G = \langle g^{-1}xg \mid g \in G\rangle$) of FC-group G is finite for all $x \in G$.

Theorem 4.4. *Let R be a ring with identity. If $G = G(R)$ is an FC-group, then $G = A \rtimes B$ is a locally normal group with finite commutant, moreover, the subgroup B is finite, $|G : Z(G)| < \infty$ and $B \cap Z(G) = 1$.*

Proof. Let $G = G(R) = A \rtimes B$ be an FC-group. Then for all element $g \in G$ the quotient-group $G/C_G(g^G)$ is finite. Lemma 4.1 implies that subgroup B is finite. By Lemma 3.10 [20] $|G : Z(G)| < \infty$ and by theorem of Baer the commutant G' is finite.

Corollary 4.5. *Let $K[H]$ be a group algebra of a group H over a field K. Then $G(K[H])$ is an FC-group if and only if the algebra $K[H]$ is finite.*

Proof. Taking into account that the groups H and K^* can be embedded into the adjoint group $(K[H])^\circ$, we see that H and K^* are finite by Theorem 4.4. Therefore, the algebra $K[H]$ is finite as well. The converse is trivial.

5. Rings with nilpotent associated groups.

Lemma 5.1. *Let T be a skew field. Then $G(T)$ is a nilpotent group if and only if $T \cong GF(2)$.*

Proof. ($\Leftarrow$) is obvious.

($\Rightarrow$). If the associated group $G(T)$ is nilpotent, then T is a field of characteristic p for some prime p. Since the field $GF(p)$ embeds in T and by Lemma 2.2 we have $|GF(p)| = p - 1 = 1$, so $p = 2$. Let $p \cong GF(2)$ be a prime subfield of T, then Exercise 9 [19] implies that that $T \supseteq P$ is a finite algebraic extension and $T = P$.

Remark 5.2.

$$U(\mathbb{Z}_{2^n}) \cong \begin{cases} 1, & n = 1; \\ \mathbb{Z}_2, & n = 2; \\ \mathbb{Z}_2 \times \mathbb{Z}_{2^{n-2}}, & n \geqslant 3. \end{cases} \quad (5.1)$$

The equation above implies that $G(\mathbb{Z}_{2^n})$ is a nilpotent 2-group.

Remark 5.3. *If p is an odd prime and $n \in \mathbb{N}$, then*

$$U(\mathbb{Z}_{p^n}) \cong \mathbb{Z}_{p^{n-1}(p-1)}. \quad (5.2)$$

From Lemma 2.2 (vii) it follows that the group $G(\mathbb{Z}_{p^n})$ is not nilpotent.

Lemma 5.4. *Let R be a ring with identity e and suppose that R has no zero divisors. Then $G(R)$ is a nilpotent group if and only if $char R = 2$ and $R^\circ = \{0\}$.*

Proof. Let $G(R) = A \rtimes B$ be a nilpotent group. Then $C_A(B) \neq 1$ by Proposition 1.6 [20]. According to Lemma 2.2 (vii), B is an identity group and consequently $R^\circ = \{0\}$. Moreover, $char R = 2$. Conversely, if $R^\circ = \{0\}$, then $G(R) \cong R^\circ$ is an abelian group. The lemma is proved.

Below $\mathcal{N}(R)$ will denote the set of all nilpotent elements of a ring R.

Theorem 5.5. *Let R be a ring with identity e. If the associated group $G(R)$ is nilpotent, then $char R = 2^m$ ($m \in \mathbb{N}$). If, thereto, ring R is a commutative, then $R^\circ = \mathcal{N}(R)$.*

Proof. Let additive order $|e|_+ = m$ for some $m \in \mathbb{N} \cup \{0\}$, then the group $G(\mathbb{Z}_m)$ is embedded in $G(R)$ (where $\mathbb{Z}_0 = \mathbb{Z}$). According to Lemma 5.4 $m \neq 0$. If $m = 2^a p_1^{a_1} \dots p_l^{a_l}$ is a canonical decomposition of m, then by Theorem 3 [19]

$$U(\mathbb{Z}_m) \cong U(\mathbb{Z}_{2^a}) \times U(\mathbb{Z}_{p_1}^{a_1}) \times \dots \times U(\mathbb{Z}_{p_l}^{a^l}), \quad (5.3)$$

where $U(\mathbb{Z}_{p_i}^{a_i}) \cong \mathbb{Z}_{p_i^{a_i-1}(p_i-1)}$ and $U(\mathbb{Z}_{2^a})$ is described in Remark 5.2. Remark 5.3 implies $a_1 = \dots = a_l = 0$ and $m = 2^a$.

Let $\bar{R} = R/2R$. If a torsion part $T(\bar{R}^\circ) \neq \{0\}$ then by Lemma 2.2 (vii), $T(\bar{R}^\circ)$ is a 2-group and therefore $T(\bar{R}^\circ) \subset \mathcal{N}(\bar{R})$. Conversely, let $\bar{x} \in \mathcal{N}(\bar{R})$, then $\bar{x}^n = \bar{0}$ for some $n \in \mathbb{N}$. It follows, that the adjoint power $\bar{x}^{(2^s)} = \bar{0}$, where $s \in \mathbb{N}$ is such that $n \leqslant 2^s$. Hence $T(\bar{R}^\circ) = \mathcal{N}(\bar{R})$.

Suppose R is a commutative ring. Then, clearly, $\mathcal{N}(\bar{R})$ is an ideal of R. Let $G(D) = \bar{A} \rtimes \bar{B}$ is a group associated with a ring $D = R/\mathcal{N}(R)$, then $\bar{B}$ is torsion free and $C_{\bar{B}}(\bar{A}) = \bar{1}$. This means, that $\bar{B}$ is embedded in the group $Aut(\bar{A})$ of the subgroup $\bar{A}$.

If $\bar{B}$ is not identity subgroup, then $[\bar{A}, \bar{B}] = \bar{A}$. It contradict to the nilpotency of the group $G(D)$. Hence $\bar{B}$ is an identity subgroup and $R^\circ = \mathcal{N}(R)$. The theorem is proved.

Remark 5.6. *Let $R = \mathbb{Q}[a]$, where $a^2 = 0$. Then R is a local Artinian ring. From the results in [21] we have $R = B + J(R)$, where the field $B = \mathbb{Q}$. It follows that $R^\circ = B^\circ \times J(R)^\circ$ is a mixed abelian group. Assume $(a, 0)$ is non zero element of $G(R)$, then*

$$(a,0)^{-1}(0,-2)(a,0) = (-a,0)(0,-2)(a,0) = (-2a,-2) \notin T(G(R)). \quad (5.4)$$

Since $(0,-2) \in T(G(R))$, *then* $G(R)$ *is a nilpotent group.*

Remark 5.7. *Let* $F = GF(p^n), n \geqslant 2$ *and* σ *is the Frobenius automorphism of the field* F. *Suppose* $F[x,\sigma]$ *is a skew polynomial algebra such that* $xa = \sigma(a)x$ *for all* $a \in F$. *Then* $R = F[x,\sigma]/(x^2)$ *is a local Artinian ring. Since* $R = J(R) + B$, *where field* $B \cong F$, *then* $U(R) \cong (1+J(R)) \rtimes B^*$, *where* $1+J(R)$ *is a p-group,* $|B^*| = p^n - 1$. *As a corollary of [11] we have that the group* $U(R)$ *is not nilpotent.*

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ПРО АСОЦІЙОВАНІ ГРУПИ КІЛЕЦЬ З УМОВАМИ СКІНЧЕННОСТІ

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Розглянуто конструкцію асоційованої групи кільця з одиницею. Охарактеризовано кільця з періодичною, FC-групою, нільпотентною асоційованими групами. Показано, що з умов скінченності для асоційованої групи $G(R)$ випливають певні умови скінченності чи комутативність кільця R.

Ключові слова: асоційована група кільця, приєднана група, FC-група, періодична група.

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