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TRIPLEABILITY OF THE CATEGORY OF (STRONGLY) SEMICONVEX COMPACTA OVER THE CATEGORY OF COMPACTA

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The notion of (strongly) semiconvex compactum and semiconvex combination generalizes a notion of convex compactum and convex combination (a "segment" that connects a point with itself is allowed to be a non-trivial loop). It is proved that a quotient space of a (strongly) semiconvex compactum for an equivalence relation closed under semiconvex combination is a (strongly) semiconvex compactum as well. Also tripleability of the category of (strongly) semiconvex compacta over the category of compacta is established.

Key words: compactum, (strongly) semiconvex compactum, left adjoint functor, tripleability.

First recall some definitions and facts from [5]. We use the following terminology and denotations : $I = [0,1]$ is *a unit segment*, *a compactum* is a (not necessarily metrizable) (bi)compact Hausdorff topological space. *A semiconvex compactum* is a compactum X with a continuous ternary operation $c : X \times X \times I \to X$ (we usually call it semiconvex combination and write $\lambda(x,y)$ instead of $c(x,y,\lambda)$) satisfying the following axioms:

1) for all $x, y \in X$, $\lambda \in I$: $\lambda(x,y) = (1-\lambda)(y,x)$ (commutative law);

2) for all $x,y,z \in X$, $\lambda,\mu,\nu \in I$, $\lambda+\mu+\nu = 1$, $\mu \neq 0$:

$$\lambda(x, \frac{\mu}{\mu+\lambda}(y,z)) = (\lambda+\mu)(\frac{\lambda}{\lambda+\mu}(x,y), z)$$

(associative law);

3) for all $x, y \in X$: $1(x,y) = x$.

4) there exists a base β of a unique uniformity inducing the topology on X [2] such that $B \in \beta$, $(x,y),(z,t) \in B$, $\lambda \in I$ implies $(\lambda(x,z), \lambda(y,t)) \in B$.

The last axiom is equivalent to the following :

4') the topology on X is generated by a saturated family of pseudometrics [2] $(d_\alpha)_{\alpha\in\mathcal{A}}$ such that $x,y,z,t \in X$, $\epsilon > 0$, $\alpha \in \mathcal{A}$, $d_\alpha(x,y) < \epsilon$, $d_\alpha(z,t) < \epsilon$, $\lambda \in I$ implies $d_\alpha(\lambda(x,z), \lambda(y,t)) < \epsilon$.

Extend the notion of semiconvex combination onto a finite number of elements of X. Let $\Delta_{n-1} = \{(\lambda_0, \ldots, \lambda_n) \in \mathbb{R}^{n+1} : \lambda_0, \ldots, \lambda_n \geqslant 0,\ \lambda_0 + \cdots + \lambda_n = 1\}$ denote the standard n-dimensional simplex. Given $(\lambda_0, \ldots, \lambda_n) \in \Delta_n$ and points $x_0, \ldots, x_n \in X$ let

$$(\lambda_0, \ldots, \lambda_n)(x_0, \ldots, x_n) = \begin{cases} x_0, & \text{if } \lambda_0 = 1; \\ \lambda_1(x_0, (\frac{\lambda_1}{1-\lambda_0}, \ldots, \frac{\lambda_n}{1-\lambda_0})(x_1, \ldots, x_n)), & \text{if } \lambda_0 \neq 1. \end{cases}$$

If *arguments* $x_0, \ldots, x_n$ of semiconvex combination are permutted simultaneously with the respective *coefficients* $\lambda_0, \ldots, \lambda_n$, the value of semiconvex combinations does not change.

For any subset $A \subset X$ the set

$$\mathrm{Cl}\{(\lambda_0, \ldots, \lambda_n)(x_0, \ldots, x_n) \mid n \in \mathbb{N},\ x_0, \ldots, x_n \in A,\ (\lambda_0, \ldots, \lambda_n) \in \Delta_n\}$$

is the least closed subset in X that contains A and is closed under semiconvex combination. It is called *the semiconvex hull* of A.

There exists the largest closed under semiconvex combination closed subset $A \subset X$ such that $\lambda : A^2 \to A$ is surjective for any $\lambda \in I$. It is called *the weak center* of X and denoted $WCtr(X)$. *The center* $Ctr(X)$ of the semiconvex compactum X is a closed subset consisting of all points $b \in X$ such that $\lambda(b, b) = b$ for any $\lambda \in I$. Always $Ctr(X) \subset WCtr(X)$, and

$$WCtr(X) = \bigcap\{(\lambda_0, \ldots, \lambda_n)(x_0, \ldots, x_n) \mid n \in \mathbb{N},\ (\lambda_0, \ldots, \lambda_n) \in \Delta_n,\ x_0, \ldots, x_n \in X\}$$

$$Ctr(X) = \bigcap\{(\lambda_1, \ldots, \lambda_n)(x, \ldots, x) \mid n \in \mathbb{N},\ (\lambda_0, \ldots, \lambda_n) \in \Delta_n,\ x \in X\}.$$

The center of X is closed under semiconvex combination and with operation induced becomes a convex compactum. Always $Ctr(X) \subset WCtr(X)$. If $Ctr(X) = WCtr(X)$, then X is called *a strongly semiconvex compactum.* Here is an alternative definition : X is a strongly semiconvex compactum if and only if for any $x \in X$ the point $(\lambda_0, \ldots, \lambda_n)(x, \ldots, x)$ converges to a unique point $y \in X$, when $(\lambda_0, \ldots, \lambda_n) \in \Delta_n$ and $\max(\lambda_1, \ldots, \lambda_n) \to 0$. This implies that if $f : X \to Y$ is a surjective map of strongly semiconvex compacta that preserves semiconvex combination (i.e., $f(\lambda(x_1, x_2)) = \lambda(f(x_1), f(x_2))$ for any $x_1, x_2 \in X$, $\lambda \in I$), and X is strongly semiconvex, then Y is strongly semiconvex as well.

For proofs see [5].

Theorem 1. *Let X be a (strongly) semiconvex compactum and "$\sim$"$\subset X \times X$ be a closed equivalence relation that is closed under semiconvex combination. If by $[x]$ the equivalence class that contains $x \in X$, is denoted, then the formula $\lambda([x], [x']) = [\lambda(x, x')]$, $x, x' \in X$, $\lambda \in I$, correctly defines an operation $Y \times Y \times I \to Y$ on $Y = X/_\sim$ such that Y is a (strongly) semiconvex compactum.*

Proof. Since "$\sim$" is closed, $X/_\sim$ is a compactum [2]. Denote by $q : X \to X/_\sim$ the quotient map. Let $x_1 \sim x_1'$, $x_2 \sim x_2'$, $x_1, x_1', x_2, x_2' \in X$, $\lambda \in I$. Then by the assumption of the theorem $\lambda(x_1, x_2) \sim \lambda(x_1', x_2')$, and the operation is well defined. Axioms (1)–(3) for Y are easy consequences of (1)–(3) for semiconvex combination in

X. Since q is a surjective continuous map of compacta, the diagram

$$\begin{array}{ccc} X \times X \times I & \xrightarrow{\text{semiconvex combination}} & X \\ {\scriptstyle q\times q\times 1_I}\downarrow & & \downarrow{\scriptstyle q} \\ (X/_\sim) \times (X/_\sim) \times I & \xrightarrow[\text{new operation}]{} & X/_\sim \end{array}$$

shows that new defined operation is continuous.

Denote by $\exp Z$ [3] the set of all nonempty closed subsets of an arbitrary compactum Z. Then the multivalued map $q^{-1}: Y \to \exp X$ is *upper semicontinuous*, i.e., for any open set $U \subset X$ the set $\{y \in Y \mid q^{-1}(y) \subset U\}$ is open. Thus for any closed $F \subset X$ the set $\{y \in Y \mid q^{-1}(y) \cap U \neq \emptyset\}$ is closed. It is easy to prove that the map $Q = (\times) \circ (q^{-1} \times q^{-1}) : Y \times Y \to \exp(X \times X)$, $Q(y_1, y_2) = d^{-1}(y_1) \times d^{-1}(y_2)$, is upper semicontinuous as well.

Take a saturated family $(\rho_\alpha)_{\alpha \in \mathcal{A}}$ of pseudometrics that generates the topology on X and satisfies (4'). For any $\alpha \in \mathcal{A}$ the formula

$$\tilde{\rho}_\alpha((x_1, x_2), (x_3, x_4)) = \max\{\rho_\alpha(x_1, x_3), \rho_\alpha(x_2, x_4)\}$$

defines a continuous pseudometric on $X \times X$. For each $\epsilon > 0$ the set

$$F_\epsilon^\alpha = \{(x_1, x_2) \in X^2 \mid \tilde{\rho}_\alpha((x_1, x_2), \text{“}\sim\text{”}) \leqslant \epsilon\} = \\ \{(x_1, x_2) \in X^2 \mid \exists z_1, z_2 \in X : z_1 \sim z_2, \rho_\alpha(x_1, z_1) \leqslant \epsilon, \rho_\alpha(x_2, z_2) \leqslant \epsilon\}$$

is closed, as well as the set

$$V_\epsilon^\alpha = \{(y_1, y_2) \in Y^2 \mid Q(y_1, y_2) \cap F_\epsilon^\alpha \neq \emptyset\} = \{(y_1, y_2) \in Y^2 \mid \exists x_1 \in q^{-1}(y_1), \\ \exists x_2 \in q^{-1}(y_2), \exists z_1, z_2 \in X : z_1 \sim z_2, \rho_\alpha(x_1, z_1) \leqslant \epsilon, \rho_\alpha(x_2, z_2) \leqslant \epsilon\}.$$

Since $(\rho_\alpha)_{\alpha \in \mathcal{A}}$ is saturated, the family $(F_\epsilon^\alpha)_{\alpha \in \mathcal{A}, \epsilon > 0}$ is a centered system of nonempty closed subsets of $X \times X$, and $\bigcap_{\alpha \in \mathcal{A}, \epsilon > 0} F_\epsilon^\alpha = \text{“}\sim\text{”}$. Suppose that $(y_1, y_2) \in \bigcap_{\alpha \in \mathcal{A}, \epsilon > 0} V_\epsilon^\alpha$. Then $\{Q(y_1, y_2) \cap F_\epsilon^\alpha \mid \alpha \in \mathcal{A}, \epsilon > 0\}$ is a centered system of nonempty closed subsets of $X \times X$. Thus its intersection is nonempty, and $Q(y_1, y_2) \cap \bigcap_{\alpha \in \mathcal{A}, \epsilon > 0} F_\epsilon^\alpha \neq \emptyset \implies Q(y_1, y_2) \cap \text{“}\sim\text{”} \neq \emptyset \implies y_1 = y_2$. Therefore we have $\bigcap_{\alpha \in \mathcal{A}, \epsilon > 0} V_\epsilon^\alpha = \Delta = \{(y, y) \mid y \in Y\}$. Obviously $V_\epsilon^\alpha \supset \Delta$ for any $\alpha \in \mathcal{A}$, $\epsilon > 0$. Moreover, $\operatorname{Int} V_\epsilon^\alpha \supset \Delta$ for any $\alpha \in \mathcal{A}$, $\epsilon > 0$. This follows from the inclusion

$$V_\epsilon^\alpha \supset U_\epsilon^\alpha = \{(y_1, y_2) \in Y^2 \mid \exists y_0 \in Y\ \forall x_1 \in q^{-1}(y_1), \forall x_2 \in q^{-1}(y_2), \\ \exists z_1, z_2 \in q^{-1}(y_0) : \rho_\alpha(x_1, z_1) < \epsilon, \rho_\alpha(x_2, z_2) < \epsilon\}.$$

The upper semicontinuity of Q implies the openness of U_ϵ^α. Obviously $U_\epsilon^\alpha \supset \Delta$. Thus $(V_\epsilon^\alpha)_{\alpha \in \mathcal{A}, \epsilon > 0}$ is a base of a unique uniformity that generates the topology on Y.

Suppose that $(y_1, y_2), (y_1', y_2') \in V_\epsilon^\alpha$, $\lambda \in I$. Then there exist $x_1, x_2, z_1, z_2, x_1', x_2', z_1', z_2' \in X$ such that $q(x_1) = y_1$, $q(x_2) = y_2$, $z_1 \sim z_2$, $\rho_\alpha(x_1, z_1) \leqslant \epsilon$, $\rho_\alpha(x_2, z_2) \leqslant \epsilon$, $q(x_1') = y_1'$, $q(x_2') = y_2'$, $z_1' \sim z_2'$, $\rho_\alpha(x_1', z_1') \leqslant \epsilon$, $\rho_\alpha(x_2', z_2') \leqslant \epsilon$. Then $\rho_\alpha(\lambda(x_1, x_1'), \lambda(z_1, z_1')) \leqslant \epsilon$, $\rho_\alpha(\lambda(x_2, x_2'), \lambda(z_2, z_2')) \leqslant \epsilon$, $\lambda(z_1, z_1') \sim \lambda(z_2, z_2')$. As $q(\lambda(x_1, x_1')) = \lambda(y_1, y_1')$, $q(\lambda(x_2, x_2')) = \lambda(y_2, y_2')$, we obtain $(\lambda(y_1, y_1'), \lambda(y_2, y_2')) \in V_\epsilon^\alpha$.

Thus $(V^\alpha_\varepsilon)_{\alpha\in\mathcal{A},\varepsilon>0}$ satisfies (4), $Y = X/_\sim$ is a semiconvex compactum and $q : X \to X/_\sim$ preserves semiconvex combination. Therefore if X is strongly semiconvex, then Y is strongly semiconvex as well.

Semiconvex compacta and their continuous mappings which preserve semiconvex combination form a category denoted by $\mathcal{S}Conv$. Strongly semiconvex compacta form a full subcategory $\mathcal{S}sConv \subset \mathcal{S}Conv$.

By $\mathcal{C}omp$ the category of compacta is denoted. Let $U : \mathcal{S}sConv \to \mathcal{C}omp$ and $U' : \mathcal{S}Conv \to \mathcal{C}omp$ be the forgetful functors.

Recall that a functor $L : \mathcal{B} \to \mathcal{C}$ is called *left adjoint* [1] to a functor $R : \mathcal{C} \to \mathcal{B}$ if there are given bijections $\theta(X, Y)$ between arrows from LX to Y in $\mathcal{B}$ and arrows from X to RY in $\mathcal{C}$ for all $X \in Ob\,\mathcal{C}$, $Y \in Ob\,\mathcal{B}$, and these bijections are natural by both arguments, i.e., the diagram

$$\begin{array}{ccc} \mathcal{B}(LX,Y) & \xrightarrow{\theta(X,Y)} & \mathcal{C}(X,RY) \\ \Big\downarrow{\scriptstyle \mathcal{C}(Lf,g)} & & {\scriptstyle \mathcal{C}(f,Rg)}\Big\downarrow \\ \mathcal{B}(LX',Y') & \xrightarrow{\theta(X',Y')} & \mathcal{C}(X',RY') \end{array}$$

commutes for any $X, X' \in Ob\,\mathcal{C}$, $Y, Y' \in Ob\,\mathcal{B}$, $f : X' \to X$, $g : Y \to Y'$.

Theorem 2. *There exist left adjoints to U and U'.*

Proof. An explicit construction of a left adjoint to U was described in [5]. Now an independent proof suitable for both cases will be given. Due to Freyd General Adjoint Functor Theorem [1] for a category $\mathcal{B}$ with all limits and a functor $R : \mathcal{B} \to \mathcal{C}$ the existence of a left adjoint $L : \mathcal{C} \to \mathcal{B}$ is equivalent to the following :
1) R preserves all limits;
2) R satisfies the *solution set condition*, i.e., for any $X \in Ob\,\mathcal{C}$ there exists a set $S \subset \{(Y, f) \mid Y \in Ob\,\mathcal{B},\ f : X \to RY\}$ (solution set) such that for any arrow $g : X \to RZ$, $Z \in Ob\,\mathcal{B}$, there is a pair $(Y, f) \in S$ and an arrow $h : Y \to Z$ in $\mathcal{B}$ such that $g = Rh \circ f$.
It suffices to check the existence of limits in $\mathcal{S}Conv$ and $\mathcal{S}sConv$ and their preservation by U' and U for two partial cases : for products and pairwise equalisers.

If X_α, $\alpha \in \mathcal{A}$ are (strongly) semiconvex compacta, then their product in $\mathcal{S}Conv$ ($\mathcal{S}sConv$) is merely a topological product with semiconvex combination defined by a formula

$$\lambda((x_\alpha), (y_\alpha)) = (\lambda(x_\alpha, y_\alpha)), \quad (x_\alpha), (y_\alpha) \in \prod_{\alpha\in\mathcal{A}} X_\alpha, \lambda \in I.$$

Clearly it is preserved by the forgetful functor.

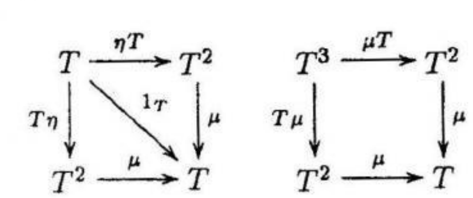
If $f, g : X \to Y$ is a parallel pair in $\mathcal{S}Conv$ of $\mathcal{S}sConv$, then its equaliser in $\mathcal{C}omp$ is a set $X_0 = \{x \in X \mid f(x) = g(x)\}$ with the embedding $i : X_0 \to X$. This set is closed in X and closed under semiconvex combination. Therefore X_0 with the restriction of semiconvex combination from X and $i : X_0 \to X$ is the equaliser of f, g in $\mathcal{S}Conv$ ($\mathcal{S}sConv$) that is preserved by U' (respectively by U).

Prove that the solution set condition holds. Suppose that Z is a (strongly) semiconvex compactum, $g : X \to Z$ is a continuous map of compacta and $|X| \leqslant \tau$, τ is infinite. Then cardinality of the set

$$\{(\lambda_0,\dots,\lambda_n)(f(x_0),\dots,f(x_n)) \mid n\in\mathbb{N},\ x_0,\dots,x_n\in X,\ (\lambda_0,\dots,\lambda_n)\in\mathbb{Q}^{n+1}\cap\Delta_n\}$$

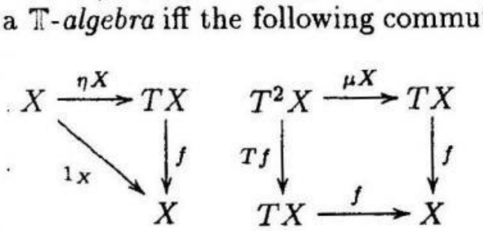
is not greater than τ. Its closure Y is the semiconvex hull of $f(X)$ in Z, and $g = h\circ f$, $h : Y \hookrightarrow Z$ is the embedding, $f \equiv g$, $f : X \to Y$. Therefore we can put S to be the set of all continuous maps from X to "representatives" of all (strongly) semiconvex compacta with density not greater than τ.

Any adjunction between $L : \mathcal{C} \to \mathcal{B}$ any $R : \mathcal{B} \to \mathcal{C}$ is uniquely determined by a pair of natural transformations [1] $\eta 1_{\mathcal{C}} \to RL$ (the *unit* of adjunction) and $\epsilon : LR \to 1_{\mathcal{B}}$ (the *counit*) such that $R\epsilon\circ\eta R = 1_R$, $\epsilon L\circ L\eta = 1_L$. Then the functor $T = RL : \mathcal{C}\to\mathcal{C}$ and natural transformations η and $\mu = R\epsilon L : T^2 \to T$ form a *triple* $\mathbb{T} = (T,\eta,\mu)$. This means that diagrams

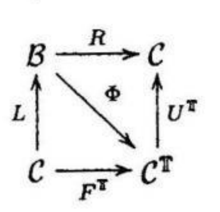


commute. Then η is called the *unit* and μ the *multiplication* of $\mathbb{T}$.

For an arbitrary triple $\mathbb{T} = (T,\eta,\mu)$ in $\mathcal{C}$ a pair (X,f), where $f : TX \to X$ is a morphism in $\mathcal{C}$, is called a $\mathbb{T}$-*algebra* iff the following commute:



An arrow $\phi : X \to Y$ is called *a map of algebras* $(X,f)\to(Y,g)$ if and only if $g\circ T\phi = \phi\circ f$. Algebras of a triple $\mathbb{T}$ in $\mathcal{C}$ and their maps form a category $\mathcal{C}^{\mathbb{T}}$. There exists a pair of adjoint functors $F^{\mathbb{T}} : \mathcal{C}\to\mathcal{C}^{\mathbb{T}}$ and $U^{\mathbb{T}} : \mathcal{C}^{\mathbb{T}}\to\mathcal{C}$, $F^{\mathbb{T}}X = (TX,\mu X)$, $F^{\mathbb{T}}\phi = T\phi$, $U^{\mathbb{T}}(X,f) = X$, $U^{\mathbb{T}}\phi = \phi$. The triple $\mathbb{T}$ arises from this pair in a way discribed above as well as from original pair L, R. There exist the unique functor (*Eilenberg-Moore comparison functor*) $\Phi : \mathcal{B}\to\mathcal{C}^{\mathbb{T}}$ that makes the diagram



commutative. If Φ is an equivalence of the categories then $\mathcal{B}$ is said to be *tripleable* [1] over $\mathcal{C}$ (with implicit adjoint functors L and R). T. Świrszcz [4] proved that convex compacta are tripleable over compacta (the left adjoint is a probability measure functor). Here is a counterpart for (strongly) semiconvex compacta.

Theorem 3. *Forgetful functors $U' : SConv \to Comp$ and $U : SsConv \to Comp$ are tripleable.*

Proof. Prove the statement for $U' : \mathcal{S}Conv \to \mathcal{C}omp$ (the case of $U : \mathcal{S}sConv \to \mathcal{C}omp$ is quite analogous). Due to Beck's precise tripleability theorem [1] it is sufficient to prove that
1) U' has a left adjoint;
2) U' reflects isomorphisms;
3) $\mathcal{S}sConv$ has and U' preserves coequalizers of U'-contractible coequaliser pairs [1].
(1) is proved above, (2) follows from the fact that isomorphisms in $\mathcal{S}Conv$ are homeomorphisms that preserve semiconvex combination. Let us prove (3).

Let $f_0, f_1 : X \to Y$ be an U'-contractible coequaliser pair in $\mathcal{S}sConv$, i.e. there exists an arrow $t : Y \to X$ in $\mathcal{C}omp$ such that $f_0 \circ t = 1_Y$, $f_1 \circ t \circ f_0 = f_1 \circ t \circ f_1$, and the pair f_0, f_1 has a coequaliser $h : Y \to Z$ in $\mathcal{C}omp$. Then h is the quotient map of the closed equivalence relation : $y_1 \sim y_2$ for $y_1, y_2 \in Y$ if and only if there exist $x_1, x_2 \in X$ such that $f_0(x_1) = f_0(x_2)$, $y_1 = f_1(x_1)$, $y_2 = f_1(x_2)$. Moreover "$\sim$" is closed under semiconvex combinations. Suppose $y_1 \sim y_1'$, $y_2 \sim y_2'$, $\lambda \in I$. Then there exist $x_1, x_1', x_2, x_2' \in X$ such that $f_0(x_1) = f_0(x_1')$, $y_1 = f_1(x_1)$, $y_1' = f_1(x_1')$, $f_0(x_2) = f_0(x_2')$, $y_2 = f_1(x_2)$, $y_2' = f_1(x_2')$. Put $x = \lambda(x_1, x_2)$, $x' = \lambda(x_1', x_2')$, $y = \lambda(y_1, y_2)$, $y' = \lambda(y_1', y_2')$. Thus $f_0(x) = f_0(x')$, $y = f_1(x)$, $y' = f_1(x')$ implies $\lambda(y_1, y_2) \sim \lambda(y_1', y_2')$. By the first theorem $X/_\sim$ is semiconvex, and h is a coequaliser of f_0, f_1 in $\mathcal{S}Conv$.

Remark. In fact we have proved that U' and U form [1] coequalizers of U'-contractible (resp. U-contractible) coequaliser pairs. Thus the comparison functors are *isomorphisms* of categories.

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МОНАДИЗОВНІСТЬ КАТЕГОРІЇ (СТРОГО) НАПІВОПУКЛИХ КОМПАКТІВ НАД КАТЕГОРІЄЮ ОПУКЛИХ КОМПАКТІВ

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Поняття (строго) напівопуклого компакта і напівопуклої комбінації узагальнюють поняття опуклого компакта та опуклої комбінації (з різницею, що "відрізок", який з'єднує точку з собою, може бути нетривіальною петлею). Доведено, що фактор-простір (строго) напівопуклого компакта є (строго) напівопуклим за умови, що відповідне відношення еквівалентності замкнене стосовно формування напівопуклих комбінацій. Також доведено монадизовність категорії (строго) напівопуклих компактів над категорією компактів.

Ключові слова: компакт, (сильно) напівопуклий компакт, лівий спряжений функтор, монадичність.

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