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ON DECOMPOSITION OF COMPLETE LINEAR GROUP INTO PRODUCT OF SOME ITS SUBGROUPS

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The group G_Φ of invertible matrices quasicommuting with the diagonal matrix Φ is considered. It is shown that the complete linear group over some Bezout domain decomposes into the product of G_Φ, lower, and upper unitriangulars groups. Nessesary and suffisient conditions for the equolity $GL(n,R) = G_\Psi^T G_\Phi$, where T denotes the transposition, are obtained. Some applications of these results are considered.

Key words: complete linear group, decomposition, subgroup, divisor of matrices.

Let R be a commutative Bezout domain in which for all $a, b, c \in R$ with $(a,b,c) = 1$, $c \neq 0$, there exists element $r \in R$, such that $(a + rb, c) = 1$. As an example of such rings one can consider the Euclidean rings, principal ideal rings, adequate rings. Let $\Phi = \operatorname{diag}(\varphi_1, \dots, \varphi_n)$ be a nonsingular d-matrix, i.e. a matrix in which $\varphi_i \mid \varphi_{i+1}$, $i = 1, \dots, n-1$. We will consider a group of matrices

$$G_\Phi = \{H \in GL_n(R) \mid H\Phi = \Phi S, \quad S \in GL_(n,R)\},$$

which consist of all invertible matrices of the form $\|h_{ij}\|_1^n$, where $h_{ij} = \frac{\varphi_i}{\varphi_j} k_{ij}$, $i = 2, \dots, n$, $j = 1, \dots, n-1$, $i > j$. In the papers [1, 2, 3] it was shown that the group G_Φ play the main role in the description of the nonassociative divisors of matrices. This paper is devoted to an investigation of this group. Let $U_{up}(n,R)$ and $U_{lw}(n,R)$ be groups of upper and lower $n \times n$ unitriangular matrices over R, respectively.

Theorem 1. $GL(n,R) = G_\Phi U_{lw}(n,R) U_{up}(n,R)$.

In order to prove this Theorem we establish a series of facts.

Lemma 1. *Let $A \in GL(n-1,R)$, $a = \| a_1 \ \dots \ a_{n-1} \|^T$ then there exists a column $x = \| x_1 \ \dots \ x_{n-1} \|^T$ such that*

$$\begin{Vmatrix} 1 & 0 \\ a & E_{n-1} \end{Vmatrix} \begin{Vmatrix} 1 & 0 \\ 0 & A \end{Vmatrix} = \begin{Vmatrix} 1 & 0 \\ 0 & A \end{Vmatrix} \begin{Vmatrix} 1 & 0 \\ x & E_{n-1} \end{Vmatrix}.$$

Proof. It is easy to see that $x = A^{-1}a$. □

Lemma 2. *Let $\varphi \neq 0$ be any fixed element of R, $(a_1, \dots, a_n) = 1$, $(a_1, \varphi) = 1$. Then the row $\| a_1 \ \dots \ a_n \|$ can be complemented to an invertible matrix of the form*

$$\left\| \begin{matrix} a_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ 0 & 1 & 0 & \dots & 0 & u_n \\ 0 & 0 & 1 & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & 0 & u_4 \\ 0 & 0 & 0 & & 1 & u_3 \\ \varphi u_1 & 0 & 0 & \dots & 0 & u_2 \end{matrix} \right\|. \tag{1}$$

Proof. Observe that $(a_1, \varphi a_2, \dots, \varphi a_n) = 1$ and use results of paper [4], which without loss of generality can be extended to our ring, complement the row $\| a_1 \ \dots \ a_n \|$ to an invertible matrix of form (1). □

We will consider a group of matrices

$$G_\Phi^T = \{H \in GL_n(R) \mid \Phi H = S\Phi, \quad S \in GL_n(R)\},$$

which consists of all invertible matrices of the form $\|h_{ij}\|_1^n$, where $h_{ij} = \frac{\varphi_j}{\varphi_i} k_{ij}$, $i = 1, \dots, n-1$, $j = 2, \dots, n$, $i < j$.

Lemma 3. *Let $(a_1, \dots, a_n) = 1$, $n \geqslant 2$ and $\left(a_1, \frac{\varphi_2}{\varphi_1} a_2, \dots, \frac{\varphi_n}{\varphi_1} a_n\right) = \delta$. Then in the groups G_Φ, G_Φ^T there exist matrices H, L such that*

$$\| a_1 \ \dots \ a_n \| H = \| \delta \ * \ \dots \ * \|,$$

$$L \| a_1 \ \dots \ a_n \|^T = \| \delta \ * \ \dots \ * \|^T.$$

Proof. There are elements $u_1, \dots, u_n$ sush that

$$a_1 u_1 + \frac{\varphi_2}{\varphi_1} a_2 u_2 + \dots + \frac{\varphi_n}{\varphi_1} a_n u_n = \delta.$$

By property 4 from [4] the element u_1 can be chosen so that $\left(u_1, \frac{\varphi_n}{\varphi_1}\right) = 1$. Hence,

$$\left(u_1, \frac{\varphi_n}{\varphi_1} (u_2, \dots, a_n)\right) = 1.$$

Since $\frac{\varphi_i}{\varphi_1} \mid \frac{\varphi_n}{\varphi_1}$, $i = 2, \dots, n$, then $\left(u_1, \frac{\varphi_2}{\varphi_1} u_2, \dots, \frac{\varphi_n}{\varphi_1} u_n\right) \mid \left(u_1, \frac{\varphi_n}{\varphi_1} u_2, \dots, \frac{\varphi_n}{\varphi_1} u_n\right) = 1$. Consequently

$$\left(u_1, \frac{\varphi_2}{\varphi_1} u_2, \dots, \frac{\varphi_n}{\varphi_1} u_n\right) = 1.$$

By a Theorem from [5] in the group G_Φ there exists a matrix H with the first row $\left\| u_1 \ \ \frac{\varphi_2}{\varphi_1} u_2 \ \ \dots \ \ \frac{\varphi_n}{\varphi_1} u_n \right\|^T$. The second part of our assertion can be proved by analogy. □

Lemma 4. *Let A be a $k \times l$ matrix and α the greatest common divisor of all elements of this matrix. If A is a submatrix of the $n \times n$ matrix B and $k + l \geqslant n + 1$ than $\alpha \mid \det B$.*

Proof. Without loss of generality we can suppose that the matrix A is in left lower corner of the matrix $B = \|b_{ij}\|_1^n$. Hence

$$\left\| \begin{matrix} b_{s1} & \dots & b_{sl} \\ \dots & \dots & \dots \\ b_{n1} & \dots & b_{nl} \end{matrix} \right\|,$$

where $s = n - k + 1$. Since $k + l \geqslant n + 1$, we obtain $l \geqslant n - k + 1 = s$. It means that the diagonal element b_{ss} is an element of the first row of the matrix A. By Lemma from [3], $\alpha \mid \det B$. □

Proof of Theorem 1. Let be $A \in GL(2, R)$. Then $\det A = e \in U(R)$. In the group G_Φ there exists a matrix $H_1 = \operatorname{diag}(1, e^{-1})$. Denote $H_1 A = \|a_{ij}\|_1^2$. Since $(a_{11}, a_{21}) = 1$, there exist elements u_1, u_2 such that

$$u_1 a_{11} + u_2 a_{21} = 1.$$

For each element $r \in R$ we have

$$(u_1 + a_{21} r) a_{11} + (u_2 - a_{11} r) a_{21} = 1.$$

Since $(u_1, a_{21}) = 1$ we see that $\left(u_1, a_{21}, \frac{\varphi_2}{\varphi_1}\right) = 1$. Thus there exists r_0 such that

$$\left(u_1 + a_{21} r_0, \frac{\varphi_2}{\varphi_1}\right) = 1.$$

We denote by $\overline{u}_1 = u_1 + a_{21} r_0, \overline{u}_2 = u_2 + a_{11} r_0$. Then

$$\left(\overline{u}_1, \frac{\varphi_2}{\varphi_1} \overline{u}_2\right) = 1,$$

so there exist x, y such that

$$\overline{u}_1 y - \frac{\varphi_2}{\varphi_1} \overline{u}_2 x = 1.$$

It means that in the group G_Φ there exists a matrix

$$H_2 = \left\| \begin{matrix} \overline{u}_1 & \overline{u}_2 \\ \frac{\varphi_2}{\varphi_1} x & y \end{matrix} \right\|.$$

Then

$$H_2 H_1 A = \left\| \begin{matrix} 1 & a \\ b & c \end{matrix} \right\| = \left\| \begin{matrix} 1 & 0 \\ b & 1 \end{matrix} \right\| \left\| \begin{matrix} 1 & a \\ 0 & 1 \end{matrix} \right\|.$$

Therefore

$$A = H \left\| \begin{matrix} 1 & 0 \\ b & 1 \end{matrix} \right\| \left\| \begin{matrix} 1 & a \\ 0 & 1 \end{matrix} \right\|,$$

where $H = (H_2 H_1)^{-1} \in G_\Phi$. Hence the result holds for $n = 2$. Let $n \geqslant 3$, and suppose that the result is established for $k < n$. Since $(a_{11}, \dots, a_{n1}) = 1$, it follows that there exist elements $u_1, \dots, u_n$ such that

$$u_1 a_{11} + \dots + u_n a_{n1} = 1,$$

where the element u_1 satisfies the condition

$$\left(u_1, \frac{\varphi_n}{\varphi_1}\right) = 1.$$

By Lemma 2 the row $\| u_1 \quad \dots \quad u_n \|$ is complementable to an invertible matrix of form (1), where $\varphi = \frac{\varphi_n}{\varphi_1}$. It is obvious that $H \in G_\Phi$. Then

$$HA = \left\| \begin{matrix} 1 & b \\ a & A_{n-1} \end{matrix} \right\| = \left\| \begin{matrix} 1 & 0 \\ a & E_{n-1} \end{matrix} \right\| \left\| \begin{matrix} 1 & 0 \\ 0 & B_{n-1} \end{matrix} \right\| \left\| \begin{matrix} 1 & b \\ 0 & E_{n-1} \end{matrix} \right\|,$$

$a = \| a_1 \quad \dots \quad a_{n-1} \|^T$, $b = \| b_1 \quad \dots \quad b_{n-1} \|$, $B_{n-1} \in GL(n-1, R)$. By the induction hypothesis $B_{n-1} = H_{n-1}UV$, where $H_{n-1} \in G_{\Phi_1}$, $\Phi_1 = \operatorname{diag}(\varphi_2, \dots, \varphi_n)$, $U \in U_{lw}(n-1, R)$, $V \in U_{up}(n-1, R)$. By Lemma 1 there exists column $x = \| x_1 \quad \dots \quad x_{n-1} \|^T$ such that

$$\left\| \begin{matrix} 1 & 0 \\ a & E_{n-1} \end{matrix} \right\| \left\| \begin{matrix} 1 & 0 \\ 0 & H_{n-1} \end{matrix} \right\| = \left\| \begin{matrix} 1 & 0 \\ 0 & H_{n-1} \end{matrix} \right\| \left\| \begin{matrix} 1 & 0 \\ x & E_{n-1} \end{matrix} \right\|.$$

Then

$$HA = \left\| \begin{matrix} 1 & 0 \\ 0 & H_{n-1} \end{matrix} \right\| \left(\left\| \begin{matrix} 1 & 0 \\ x & E_{n-1} \end{matrix} \right\| \left\| \begin{matrix} 1 & 0 \\ 0 & U \end{matrix} \right\| \right) \left(\left\| \begin{matrix} 1 & 0 \\ 0 & V \end{matrix} \right\| \left\| \begin{matrix} 1 & b \\ 0 & E_{n-1} \end{matrix} \right\| \right) =$$

$$= \left\| \begin{matrix} 1 & 0 \\ 0 & H_{n-1} \end{matrix} \right\| \left\| \begin{matrix} 1 & 0 \\ x & U \end{matrix} \right\| \left\| \begin{matrix} 1 & b \\ 0 & V \end{matrix} \right\|.$$

Hence,

$$A = \left(H^{-1} \left\| \begin{matrix} 1 & 0 \\ 0 & H_{n-1} \end{matrix} \right\| \right) \left\| \begin{matrix} 1 & 0 \\ x & U \end{matrix} \right\| \left\| \begin{matrix} 1 & b \\ 0 & V \end{matrix} \right\|.$$

Taking into account that $\left\| \begin{matrix} 1 & 0 \\ 0 & H_{n-1} \end{matrix} \right\| \in G_\Phi$, we see that our statement is true. □

Let A be an $n \times n$ matrix over R. Since R is a commutative elementary divisor domain [6], there exist invertible matrices P and Q such that

$$PAQ = \operatorname{diag}(\varepsilon_1, \dots, \varepsilon_n) = \Psi,$$

which is a d-matrix. The matrix Ψ is named canonical diagonal form of the matrix A. Denote by $K(f)$ the set of representatives of the conjugate class of the factor-ring R/Rf, where $f \in R$. Let

$$V(\Psi, \Phi) = \{V = \|v_{ij}\|_1^n \in U_{lw}(n, R) \mid v_{ij} = \frac{\varphi_i}{(\varphi_i, \varepsilon_j)} k_{ij}, k_{ij} \in K\left(\frac{(\varphi_i, \varepsilon_j)}{\varphi_j}\right)\},$$

$i = 2, \dots, n$, $j = 1, \dots, n-1$, $i > j$.

Corollary 1. $(V(\Psi, \Phi)U_{up}(n, R)P)^{-1}\Phi$ *is the set of left divisors of the matrix A which contain all left nonassociative by right divisors of this matrix with canonical diagonal form Φ.*

Proof. We define

$$L(\Psi, \Phi) = \{L \in GL(n, R) \mid L\Psi = \Phi S, \quad S \in M(n, R)\}.$$

By Corollary 3 from [3] the set $L(\Psi, \Phi)$ consists of all invertible matrices of the form $\|l_{ij}\|_1^n$, where $l_{ij} = \frac{\varphi_i}{(\varphi_i, \varepsilon_j)} k_{ij}$, $i = 2, \ldots, n$, $j = 1, \ldots, n-1$, $i > j$. From Proposition from [3] it follows that the set $(L(\Psi, \Phi)P)^{-1}\Phi$ is the set of left divisors of the matrix A which contain all left nonassociative by right divisors of this matrix with canonical diagonal form Φ. Let $T \in V(\Psi, \Phi)$, $N \in U_{up}(n, R)$. Since $U_{up}(n, R) \subset G_\Psi$, we have

$$TN\Psi = T\Psi S_1 = \Phi S_2 S_1, S_1 \in GL(n, R), S_2 \in M(n, R).$$

Therefore $V(\Psi, \Phi)U_{up}(n, R) \subset L(\Psi, \Phi)$. Consequently, $(V(\Psi, \Phi)U_{up}(n, R)P)^{-1}\Phi$ is the set of left divisors of the matrix A with canonical diagonal form Φ. We will show that this set is contain all left nonassociative by right divisors of the matrix A with canonical diagonal form Φ.

Let $L \in L(\Psi, \Phi)$, it means that the matrix $B = (LP)^{-1}\Phi$ is the left divisor of the matrix A. By Theorem 1, $L = HUV$, where $H \in G_\Phi$, $U \in U_{lw}(n, R)$, $V \in U_{up}(n, R)$. Hence, $U = H^{-1}LV^{-1}$. Since $V^{-1} \in G_\Psi$, it follows that

$$U\Psi = H^{-1}LV^{-1}\Psi = H^{-1}L\Psi S_1 = H^{-1}\Phi S_2 S_1 = \Phi(S_3 S_2 S_1).$$

Thus $U \in L(\Psi, \Phi)$. By Lemma 3 from [7] in the group G_Φ there exists a matrix H_1 such that $H_1 U = T_1 \in L(\Psi, \Phi)$. Consequently,

$$B = (LP)^{-1}\Phi = (HUVP)^{-1}\Phi = \left(HH_1^{-1}(H_1U)VP\right)^{-1}\Phi =$$

$$= (T_1VP)^{-1}\left(H_1H^{-1}\right)^{-1}\Phi = (T_1VP)^{-1}\Phi S = B_1 S,$$

$S \in GL(n, R)$ where $B_1 = (T_1VP)^{-1}\Phi S \in (V(\Psi, \Phi)U_{up}(n, R)P)^{-1}\Phi$. It means that every left divisor of the matrix A with canonical diagonal form Φ in the set $(V(\Psi, \Phi)U_{up}(n, R)P)^{-1}\Phi$ have associative by right matrix. The proof of the Corollary is complete. □

Theorem 2. *Let* $\Phi = \operatorname{diag}(\varphi_1, \ldots, \varphi_n)$ *and* $\Psi = \operatorname{diag}(\varepsilon_1, \ldots, \varepsilon_n)$ *be nonsingular d-matrices. In order that* $GL(n, R) = G_\Psi^T G_\Phi$, *it is necessary and sufficient that* $\left(\det \frac{1}{\varphi_1}\Phi, \det \frac{1}{\varepsilon_1}\Psi\right) = 1$.

Proof. **Necessity.** Let $\left(\det \frac{1}{\varphi_1}\Phi, \det \frac{1}{\varepsilon_1}\Psi\right) = \delta$ and φ_s, ε_r are the first diagonal elements of the matrices Φ, Ψ with the property $\delta | \frac{\varphi_s}{\varphi_1}$, $\delta | \frac{\varepsilon_r}{\varepsilon_1}$, $2 \leqslant r, s \leqslant n$. Then $\delta | \frac{\varphi_i}{\varphi_1}$, $i = s, s+1, \ldots, n$ and $\delta \frac{\varepsilon_j}{\varepsilon_1}$, $j = r, r+1, \ldots, n$. Consequently, in the lower left corner of every matrix of the group G_Φ there is an $(n-s+1) \times (s-1)$ submatrix all elements of which are divisible by δ. And in the upper right corner every matrix of the group G_Ψ^T is $(r-1) \times (n-r+1)$ submatrix all elements of which are divisible by δ. Since $GL(n, R) = G_\Psi^T G_\Phi$ then

$$LH = \begin{Vmatrix} 0 & & 1 \\ & \vdots & \\ 1 & & 0 \end{Vmatrix} = T,$$

where $L \in G_\Psi^T$, $H \in G_\Phi$. Thus, $L = TH^{-1}$. Therefore, in the left upper corner of the matrix L there is an $(n-s+1) \times (s-1)$ submatrix all elements of which are divisible by δ. Taking into account structure of elements of the group G_Ψ^T we come to the conclusion that $s < r$, otherwise all elements of first row of the matrix L would be divisible by δ. Possible cases:

1. $n-s+1 \leqslant r-1$. Then the matrix L has $(n-s+1)\times((s-1)+(n-r+1))$ submatrix all elements of which are divisible by δ. Since,

$$(n-s+1)+(s-1)+(n-r+1)=(n+1)+(n-r)\geqslant n+1$$

by Lemma 4, $\delta|\det L \in U(R)$. Therefore $\delta = 1$.

2. $n-s+1 > r-1$. Then the matrix L contains an $(r-1)\times((s-1)+(n-r+1))$ submatrix all elements of which are divisible by δ. Since,

$$(r-1)+(s-1)+(n-r+1)=n+s-1=(n+1)+(s-2)\geqslant n+1,$$

as above $\delta = 1$.

Sufficiency. Let $A = \|a_{ij}\|_1^2 \in GL(2,R)$ and $\left(a_{11}, \frac{\varphi_2}{\varphi_1}a_{12}\right) = \delta$. By Lemma 3 there exists $H \in G_\Phi$ such that

$$AH = \begin{Vmatrix} \delta & b_{12} \\ b_{21} & b_{22} \end{Vmatrix}.$$

Since $\left(\frac{\varphi_2}{\varphi_1}, \frac{\varepsilon_2}{\varepsilon_1}\right) = 1$ and $\delta|\frac{\varphi_2}{\varphi_1}$, it follows that $\left(\delta, \frac{\varepsilon_2}{\varepsilon_1}\right) = 1$. Therefore, there exists $L \in G_\Psi^T$ such that $\det LAH = 1$ and

$$LAH = \begin{Vmatrix} 1 & a \\ b & c \end{Vmatrix} = \begin{Vmatrix} 1 & 0 \\ b & 1 \end{Vmatrix}\begin{Vmatrix} 1 & a \\ 0 & 1 \end{Vmatrix}.$$

Consequently,

$$A = \left(L^{-1}\begin{Vmatrix} 1 & 0 \\ b & 1 \end{Vmatrix}\right)\left(\begin{Vmatrix} 1 & a \\ 0 & 1 \end{Vmatrix}H^{-1}\right).$$

Hence the result holds for $n = 2$.

Let $n \geqslant 3$, and suppose that the result is established for $k < n$. Let $A = \|a_{ij}\|_1^n \in GL(n,R)$. By analogy we can find matrices $L \in G_\Psi^T$, $H \in G_\Phi$ such that

$$LAH = \begin{Vmatrix} 1 & \mathbf{0} \\ \mathbf{0} & A_{n-1} \end{Vmatrix}.$$

By the induction hypothesis $A_{n-1} = L_{n-1}H_{n-1}$, where $L_{n-1} \in G_{\Psi_1}^T$, $H_{n-1} \in G_{\Phi_1}$, $\Psi_1 = \operatorname{diag}(\varepsilon_2, \dots, \varepsilon_n)$, $\Phi_1 = \operatorname{diag}(\varphi_2, \dots, \varphi_n)$. Hence,

$$A = \left(L^{-1}\begin{Vmatrix} 1 & \mathbf{0} \\ \mathbf{0} & L_{n-1} \end{Vmatrix}\right)\left(\begin{Vmatrix} 1 & \mathbf{0} \\ \mathbf{0} & H_{n-1} \end{Vmatrix}H^{-1}\right).$$

Since $\begin{Vmatrix} 1 & \mathbf{0} \\ \mathbf{0} & L_{n-1} \end{Vmatrix} \in G_\Psi^T$ and $\begin{Vmatrix} 1 & \mathbf{0} \\ \mathbf{0} & H_{n-1} \end{Vmatrix} \in G_\Phi$, the proof of our statement is complete. $\square$

Corollary 2. *Let A, B be matrices with the canonical diagonal form $\Psi = \operatorname{diag}(\varepsilon_1, \dots, \varepsilon_n)$, $\Phi = \operatorname{diag}(\varphi_1, \dots, \varphi_n)$, respectively. If $\left(\det \frac{1}{\varphi_1}\Phi, \det \frac{1}{\varepsilon_1}\Psi\right) = 1$ then the matrix AB has the canonical diagonal form $\Psi\Phi$.*

Proof. Since, $A = P_A^{-1}\Psi Q_A^{-1}$, $B = P_B^{-1}\Phi Q_B^{-1}$, where $P_A, Q_A, P_B, Q_B \in GL(n,R)$, we have $AB = P_A^{-1}\Psi\left(Q_A^{-1}P_B^{-1}\right)\Phi Q_B^{-1}$. By Theorem 2, $Q_A^{-1}P_B^{-1} = UV$, where $U \in G_\Psi^T$,

$V \in G_\Phi$. Consequently,

$$AB = P_A^{-1}(\Psi U)(V\Phi)Q_B^{-1} = \left(P_A^{-1}S_1\right)\Psi\Phi\left(S_2Q_B^{-1}\right) \sim \Psi\Phi.$$

□

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ПРО РОЗКЛАД ПОВНОЇ ЛІНІЙНОЇ ГРУПИ В ДОБУТОК ДЕЯКИХ ЇЇ ПІДГРУП

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Розглянуто групу G_Φ-оборотних матриць, які квазікомутують з діагональною матрицею Φ. Показано, що над деякою областю Безу повна лінійна група розкладається в добуток G_Φ груп нижніх і верхніх унітрикутних матриць. Зазначено необхідні та достатні умови для того, щоб $GL(n,R) = G_\Psi^T G_\Phi$, де T – знак транспонування. Одержані результати використано для опису дільників матриць.

Ключові слова: повна лінійна група, розклад, підгрупа, дільники матриць.

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