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ASYMPTOTIC CATEGORY AND SPACES OF PROBABILITY MEASURES

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An example is provided of a proper metric space whose space of probability measures is not an absolute extensor for the asymptotic category in the sense of Dranishnikov.
Key words: asymptotic category, probability measures.

The functor of probability measures P in the asymptotic topology was first considered by A. Dranishnikov [D]. It is remarked in [D] that the space $P(X)$ is an absolute extensor for the class of asymptotically Lipschitz maps defined on the proper metric spaces of finite asymptotic dimension and, more generally, of slow dimension growth. Here a metric space X is said to be of *slow dimension growth* if $\lim_{L\to\infty} m(L)/L = 0$; by $m(L)$ the minimal multiplicity of a uniformly bounded cover of X with the Lebesgue number $> L$ is denoted. The problem whether the space $P(X)$ is an absolute extensor for the category of all proper metric spaces and asymptotically Lipschitz maps was formulated in [D] (Problem 12). As remarked in [D], an affirmative solution of this problem would allow to prove a homotopy extension theorem in the asymptotic category in full generality. In this paper we provide a negative solution of this problem (see Section 3).

In Section 4 we consider another problem mentioned in [D], namely that of relationship between the cone (in the sense of Dranishnikov) of a proper metric space X and the join $X * \mathbb{R}_+$. It turns out that these objects are not always isomorphic as objects of the asymptotic category (see the definition below).

This paper was finished when the author visited the University of Florida.

1. Preliminaries A typical metric will be denoted by d. A map $f\colon X \to Y$ between metric spaces is called (λ,ε)-*Lipschitz* for $\lambda > 0$, $\varepsilon \geqslant 0$ if $d(f(x), f(x')) \leqslant \lambda d(x,x') + \varepsilon$ for every $x, x' \in X$. A map is called *asymptotically Lipschitz* if it is (λ,ε)-Lipschitz for some $\lambda, \varepsilon > 0$.

The $(1,0)$-Lipschitz maps are also called *Lipschitz* or *short*. By $\mathrm{Lip}(X)$ we denote the set of all Lipschitz functions on X.

A metric space X is called *proper* if every closed ball in X is compact.

The *asymptotic category* $\mathcal{A}$ is introduced by A. Dranishnikov [D]. The objects of $\mathcal{A}$ are proper metric spaces and the morphisms are proper asymptotically Lipschitz maps.

We also need a notion of asymptotic Lipschitz equivalence, which is a weaker notion than that of isomorphism in $\mathcal{A}$. Two proper metric spaces, X and Y, are *asymptotically Lipschitz equivalent* if there exist proper asymptotically Lipshchitz maps $f: X \to Y$ and $g: Y \to X$ such that the compositions gf and fg are of finite distance (in the sup-metric) from the corresponding identity maps.

A metric space X is said to be an *absolute extensor (AE)* for $\mathcal{A}$ if for every proper asymptotically Lipshitz map $f: A \to X$ defined on a closed subset A of a proper metric space Y there is a proper asymptotically Lipshitz extension $\bar{f}: Y \to X$.

A metric space is called C-*connected*, for $C > 0$ if for every $x, y \in X$ there exists a sequence $x = z_0, z_1, \dots, z_{k-1}, z_k = y$ such that $d(z_{i-1}, z_i) \leqslant C$ for every $i = 1, \dots, k$.

Lemma. *Suppose that $f: X \to Y$ is an asymptotically Lipschitz map of proper metric spaces and X is a C-connected space, for some $C > 0$. Then there exists $C' > 0$ such that the space Y is C'-connected.*

Proof. It is easy to see that Y is C'-connected with $C' = C\lambda + s$.

A metric space X is said to be a *geodesic metric space* if for every two points $x, y \in X$ there is an isometric embedding $j: [0, d(x,y)] \to X$ (the segment $[0, d(x,y)]$ is endowed with the euclidean metric) such that $j(0) = x$ and $j(d(x,y) = y$.

The following proposition is a version of Proposition 1.4 from [D] (see also [R]).

Proposition. *Let $f: X \to Y$ be a map of metric spaces. If X is a geodesic metric space and there exists $C > 0$ such that $d(f(x), f(y)) \leqslant C$ for any $x, y \in X$ with $d(x,y) \leqslant 1$, then f is asymptotically Lipschitz.*

Proof. The proof of Proposition 1.4 from [D] also works in our situation.

1.1. Spaces of probability measures. For a metric space X let $P(X)$ denote the space of probability measures on X with compact supports. We identify the measures with the corresponding functionals on the set $C(X)$ of continuous real-valued functions on X. For $x \in X$ by δ_x we denote the Dirac measure concentrated at X. There are different metrizations of the space of probability measures (see, e. g., [H, S, Z]). Following [H] we endow the space $P(X)$ with the following metric:

$$d(\mu, \nu) = \sup\{|\mu(\varphi) - \nu(\varphi)| : \varphi \in \mathrm{Lip}(X)\}.$$

In general, the metric space $P(X)$ is not locally compact for a proper metric space X. We complete it with respect to the defined metric and preserve the denotation $P(X)$ for the completed space. However, even this complete space is not, in general, proper, as the following example shows. Let $X = \{0\} \cup \mathbb{N}$, with the standard metric. For every $n \in \mathbb{N}$ denote by μ_n the probability measure $(1 - 2^{-n})\delta_0 + 2^{-n}\delta_{2^n}$. For every $m \in \mathbb{N}$ denote by φ_m the function defined by the formula $\varphi_m(x) = \max\{0, x - 2^m\}$. Then

$$d(\mu_n, \delta_0) = \sup\{2^{-n}|\varphi(0) - \varphi(2^n)| \mid \varphi \in \mathrm{Lip}(X)\} = 1.$$

On the other hand, if $m, n \in \mathbb{N}$, $m < n$, then

$$\begin{aligned}(\mu_m, \mu_n) \geqslant & |(2^{-m} - 2^{-n})\varphi_m(0) + 2^{-m}\varphi_m(2^m) - 2^{-n}\varphi_m(2^n)| \\ = & 2^{-n}(2^n = 2^m) \geqslant 1/2,\end{aligned}$$

and therefore the set $\{\mu_n \mid n \in \mathbb{N}\}$ is a 1/2-discrete infinite subset of the 1-ball in $P(X)$ centered at δ_0.

This example also demonstrates that the spaces $P_n(X)$ of probability measures with supports of cardinality $\leqslant n$ are not objects of the category $\mathcal{A}$.

Note that this lack of local compactness for the spaces of probability measures causes some difficulties in defining the notion of convexity in the asymptotic category.

Suppose $\mu \in P(\mathbb{R}^n)$, $\mu = \sum_{i=1}^k \alpha_i \delta_{x_i}$. Put $b(\mu) = \sum_{i=1}^k \alpha_i x_i$.

1.1. Lemma. *Let $f: X \to Y$ be a short map. Then the map $P(f)$ defined by the formula $Pf(\sum_{i=1}^k \alpha_i x_i) = \sum_{i=1}^k \alpha_i f(x_i)$ is a short map from the set of probability measures with finite supports on X to $P(Y)$.*

Proof. Obvious.

The lemma allows us to extend the map $P(f)$ to a short map of $P(X)$ into $P(Y)$. We preserve the notation $P(f)$ for this extended map.

1.2. Lemma. *The map b is a short map from the set of all probability measures on $\mathbb{R}^n$ with finite supports into $\mathbb{R}^n$.*

Proof. Suppose $\mu = \sum_{i=1}^k \alpha_i \delta_{x_i}, \nu = \sum_{j=1}^l \beta_j \delta_{y_j} \in P(\mathbb{R}^n)$ and $b(\mu) \neq b(\nu)$. Denote by $p: \mathbb{R}^n \to \mathbb{R}$ the orthogonal projection onto the direction of the vector $b(\mu) - b(\nu)$. Then

$$\begin{aligned} \|b(\mu) - b(\nu)\| &= |p(b(\mu)) - p(b(\nu))| = \sum_{i=1}^k \alpha_i p(x_i) - \sum_{j=1}^l \beta_j p(y_j) \\ &= |\mu(p) - \nu(p)| \leqslant d(\mu, \nu), \end{aligned}$$

because $p \in \mathrm{Lip}(X)$.

Lemma 1.2 allows us to extend the map b to a short map from $P(\mathbb{R}^n)$ to $\mathbb{R}^n$. This extended map will be also denoted by b. The map b is called the *barycenter map*.

3. Space of probability measures which is not an absolute extensor. For every n, the euclidean space $\mathbb{R}^n$ is naturally identified with the subspace $\{(x_i) \mid x_i = 0$ for all $j > n\}$ of the space ℓ^2 of square-summable sequences.

We endow the subspace $X = \bigcup_{n \in \mathbb{N}} \{n\} \times \mathbb{R}^n \subset \mathbb{R} \times \ell^2$ with the metric

$$d(n, (x_i)), (m, (y_i)) = (|m - n|^2 + \|(x_i) - (y_i)\|^2)^{1/2}.$$

Obviously, X is a proper metric space. For every n we denote by $p_n: X \to \mathbb{R}^n$ a map defined by the formula $p_n(m, (x_i)) = (x_1, \dots, x_n)$. Clearly, p_n is a short map.

We are going to show that the space $P(X)$ is not an absolute extensor in the category $\mathcal{A}$.

It is shown in [L] (see Theorem 1.5 therein) that for any $n \geqslant 2$ there exists a metric space extension X_n of the euclidean space $\mathbb{R}^n$ such that there is no (λ, ε)-Lipschitz retraction from X_n onto $\mathbb{R}^n$ with $\lambda < n^{1/4}$. For the sake of completeness we provide the details of the construction. Following [L], for every natural k and natural $n \geqslant 2$ we define graphs $G_{n,k}$ as follows: the set of vertices $V(G_{n,k})$ is the union of $I(G_{n,k})$ and $T(G_{n,k})$, where

$$\begin{aligned} I(G_{n,k}) &= \{x = (x_1, \dots, x_n) \in \mathbb{R}^n : |x_i| = k \text{ for all } i\}, \\ T(G_{n,k}) &= \{x = (x_1, \dots, x_n) \in \mathbb{R}^n : |x_i| = 2k \text{ for all } i\}; \end{aligned}$$

the set of edges $E(G_{n,k})$ is defined by the condition: $\{x, y\} \in E(G_{n,k})$ if and only if $x, y \in V(G_{n,k})$ and either $\|x - y\| = 2k$ or $y = 2x$ (we suppose that the spaces $\mathbb{R}^n$ are endowed with the Euclidean metric).

The set $V(G_{n,k})$ is equipped with the metric $d = d_{n,k}$,

$$d(x,y) = \inf\{\sum_{i=1}^{l} \|x_{i-1}, x_i\| : (x = x_0, x_1, \dots, x_l = y) \text{ is a path in } \dot{G}_{n,k}\}.$$

Denote by Y the metric space defined as follows. Put

$$Y = \left(X \sqcup \left(\bigcup_{k=1}^{\infty} \bigcup_{n=2}^{\infty} V(G_{n,k})\right)\right) / \sim,$$

where the equivalence relation $\sim$ is defined by identification of every $x \in T(G_{n,k})$ with $x \in \mathbb{R}^n$. The metric on Y is the maximal metric that agrees with the initial metric on X and the metric $d_{n,k}$ on every $V(G_{n,k})$. It easily follows from the construction that Y is a proper metric space, i.e. an object of the category $\mathcal{A}$.

Let $f: Y \to P(X)$ be the map that sends $x \in X$ to $\delta_x \in P(X)$. The map f is an isometric embedding and we are going to show that there is no asymptotically Lipschitz extension of f onto Y. Assume the contrary and let $\bar{f}: Y \to P(X)$ be such an extension. Then there are $\lambda > 0$ and $\varepsilon > 0$ such that

$$d(\bar{f}(x), \bar{f}(x')) \leqslant \lambda d(x, x') + \varepsilon$$

for all $x, x' \in Y$.

Let $n > \lambda^4$. Since the maps $P(p_n)$ and $b: P(\mathbb{R}^n) \to \mathbb{R}^n$ are short, we conclude that the map $b \circ P(p_n) \circ \bar{f}|X_n$ is a (λ, ε)-Lipschitz retraction from X_n onto $\mathbb{R}^n$, which gives us a contradiction.

4. Cone and join. The cone construction is of importance in the asymptotic topology as it allows to apply asymptotic methods for investigation of topological and metric properties of spaces. The open cone construction of compact metric spaces is considered in [R].

In the case of noncompact spaces, the following construction of cone is proposed by A. Dranishnikov [D]. Let X be a proper metric space with base point x_0.

Denote by CX the quotient space of the subspace

$$\{(x,t) \in X \times \mathbb{R} : |t| \leqslant d(x, x_0)\} \subset X \times \mathbb{R}$$

with respect to the following equivalence relation $\sim$: $(x_1, t_1) \sim (x_2, t_2)$ if and only if either $(x_1, t_1) = (x_2, t_2)$ or $t_1 = -d(x_1, x_0) = -d(x_2, x_0) = t_2$. Denote by $[x, t] \in CX$ the equivalence class of X that contains (x, t). We endow CX with the quotient metric ϱ,

$$\begin{aligned}\varrho([x_1,t_1],[x_2,t_2]) = \inf\{\sum_{i=0}^{k} d((y_{2i}, s_{2i}), (y_{2i+1}, s_{2i+1})) :\\ (x_1,t_1) = (y_0,s_0),\ (x_2,t_2) = (y_{2k+1}, s_{2k+1})\\ \text{and } (y_{2k-1}, s_{2k-1}) \sim (y_{2k}, s_{2k}),\ i = 1,\dots,k\}.\end{aligned}$$

The obtained metric space (CX, ϱ) is called the *cone* of X.

Given two proper metric spaces X, Y with base points x_0, y_0 respectively we define their wedge $X \vee Y$ as the quotient space $(X \sqcup Y)/\{x_0, y_0\}$ endowed with the maximal metric that makes the natural embeddings $X \to X \vee Y$, $Y \to X \vee Y$ to be isometric embeddings. The subspace

$$X * Y = \{t\delta_x + (1-t)\delta_y : d(x, x_0) = d(y, y_0),\ t \in [0,1]\}$$

of $P(X \vee Y)$ is called the *join* of X and Y.

Proposition. *The space $X * \mathbb{R}_+$, up to asymptotically Lipschitz equivalence, does not depend on the choice of base point.*

Proof. Let $x_1, x_2 \in X$ be base point. Hereafter, $X \vee_i \mathbb{R}_+$ and $X *_i \mathbb{R}_+$ denote respectively the wedge and the join with respect to the base point x_i, $i = 1, 2$. Denote by φ_i the distance function to the point x_i, $i = 1, 2$. Define a map $f: X *_1 \mathbb{R}_+ \to X *_2 \mathbb{R}_+$ by the formula

$$f(t\delta_x + (1-t)\delta_{\varphi_1(x)} = t\delta_x + (1-t)\delta_{\varphi_2(x)}.$$

Obviously, f is a bijective map and it is sufficient to show that the maps f and f^{-1} are asymptotically Lipschitz. Because of similarity, we prove this only for f.

Suppose that $x, y \in X$, $t\delta_x + (1-t)\delta_{\varphi_1(x)}$, $s\delta_y + (1-s)\delta_{\varphi_1(y)} \in X *_1 \mathbb{R}_+$ and

$$d(t\delta_x + (1-t)\delta_{\varphi_1(x)},\ s\delta_y + (1-s)\delta_{\varphi_1(y)}) = K.$$

There exists a short function $\alpha_1: X \vee \mathbb{R}_+ \to \mathbb{R}$ such that

$$|t\alpha_1(x) + (1-t)\alpha_1(\varphi_1(x)) - s\alpha_1(y) - (1-s)\alpha_1(\varphi_1(y))| = K.$$

Define a function $\alpha_2: X \vee \mathbb{R}_+ \to \mathbb{R}$ by the conditions $\alpha_2|X = \alpha_1$ and $\alpha_2(r) = \alpha_1(r) - \alpha_1(x_1) + \alpha_1(x_2)$ for $r \in \mathbb{R}_+$ (we identify X and $\mathbb{R}_+$ with the subspaces of $X \vee_i \mathbb{R}_+$ along the natural embeddings). Then α_2 is a short function and we obtain

$$\begin{aligned}
&d(f(t\delta_x + (1-t)\delta_{\varphi_1(x)}), f(s\delta_y + (1-s)\delta_{\varphi_1(y)}))\\
&\leqslant |t\alpha_2(x) + (1-t)\alpha_2(\varphi_2(x)) - s\alpha_2(y) - (1-s)\alpha_2(\varphi_2(y))|\\
&\leqslant |t\alpha_1(x) + (1-t)\alpha_1(\varphi_1(x)) - s\alpha_1(y) - (1-s)\alpha_1(\varphi_1(y))|\\
&+|s-t||\alpha_1(x_2) - \alpha_1(x_1)| + (1-t)|\alpha_1(\varphi_2(x)) - \alpha_1(\varphi_1(x))|\\
&+(1-s)|\alpha_1(\varphi_2(y)) - \alpha_1(\varphi_1(y))|\\
&\leqslant K + 4d(x_2, x_1).
\end{aligned}$$

This means that f is $(1, 4d(x_2, x_1))$-Lipschitz.

A. Dranishnikov asked in [D] whether the spaces CX and $X * \mathbb{R}_+$ are asymptotically Lipschitz equivalent for every proper metric space X. The following example demonstrates that this is not the case.

Example. Let $X = \omega$ (the set of all finite ordinals) with base point 0. We endow ω with a metric d defined as follows: $d(i,j) = \max\{i,j\}$ whenever $i \neq j$. Then $CX = \{(i,t) \in \omega \times \mathbb{R} \mid |t| \leqslant i\}$. Let $A_i = \{i\} \times [-i, i]$, because the equivalence relation $\sim$ in this case is trivial. Note that $d(A_i, C\omega \setminus A_i) \geqslant i$.

We are going to show that there is no proper asymptotically Lipschitz map from $\omega * \mathbb{R}_+$ to $C\omega$. Note first that the space $\omega * \mathbb{R}_+$ is obviously 1-connected. Suppose that there exists a proper (λ, ε)-Lipschitz map $f: \omega * \mathbb{R}_+ \to C\omega$, where $\lambda, \varepsilon > 0$. Then the image $f(\omega * \mathbb{R}_+)$ is a $\lambda + \varepsilon + 1$-connected set. Suppose that $f(\omega * \mathbb{R}_+) \cap A_i \neq \emptyset$

for some $i \in \omega$, then $f(\omega * \mathbb{R}_+) \cap A_j = \emptyset$ for all $j > \max\{\lambda + s, i\}$. Therefore, the set $f(\omega * \mathbb{R}_+)$ is compact, which contradicts to the properness of f.

Nevertheless, in some cases the cone CX and the space $X * \mathbb{R}_+$ are isomorphic as objects of the category $\mathcal{A}$. The following proposition is a counterpart of Lemma 2.4 from [D].

Proposition. *The space* $\mathbb{R}^n * \mathbb{R}_+$ *is isomorphic to* $\mathbb{R}^{n+1}_+ = \{(x_1, \ldots, x_{n+1} \mid x_{n+1} \geqslant 0\}$.

Proof. Let $i: \mathbb{R}^n \vee \mathbb{R}_+ \to \mathbb{R}^{n+1}_+$ be the map acting by the formula: $i(x) = (x, 0)$, $i(t) = (0, t)$, where $x \in \mathbb{R}^n$, $t \in \mathbb{R}_+$. Denote by $f: \mathbb{R}^n * \mathbb{R}_+ \to \mathbb{R}^{n+1}_+$ the restriction of the composition

$$P(\mathbb{R}^n \vee \mathbb{R}_+) \xrightarrow{P(i)} P(\mathbb{R}^{n+1}_+) \xrightarrow{b} \mathbb{R}^{n+1}_+$$

onto the subspace $\mathbb{R}^n * \mathbb{R}_+$. Obviously, F is a short bijective map and therefore it is sufficient to prove that the map $g = f^{-1}$ is asymptotically Lipschitz. The explicit formula for g is

$$g(x, t) = \frac{\|x\|}{\|x\| + t}\delta_{\bar{x}} + \frac{t}{\|x\| + t}\delta_{\|x\|+t},$$

where $x \in \mathbb{R}^n$, $t \in \mathbb{R}_+$, $\bar{x} = \frac{x}{\|x\|}(\|x\| + t)$; if $\|x\| = 0$, then $g(x, t) = \delta_t$. Note that it is easy to verify that g is a continuous function.

Given $(x, t), (y, s) \in \mathbb{R}^{n+1}_+$ with $\|(x, t) - (y, s)\| \leqslant 1$, we suppose, without loss of generality, that $0 < \|x\| + t \leqslant \|y\| + s$. Let $h: \mathbb{R}^{n+1}_+ \to \mathbb{R}^{n+1}_+$ be the homothety map centered at 0 and with coefficient $(\|y\| + s)/(\|x\| + t)$. Obviously, $d(h|\text{supp}(g(x, t)), \text{id}) \leqslant 1$ and therefore $d(P_2(h)(g(x, t)), g(x, t)) = d(g(h(x, t)), g(x, t)) \leqslant 1$.

Put $(x_1, t_1) = h(x, t)$; we have $\|x_1\| + t_1 = \|y\| + s$.

We are going to estimate the distance between $g(x_1, t_1)$ and $g(y, s)$. Note that $\|(x_1, t_1) - (y, s)\| \leqslant 2$. Without loss of generality we may assume that $\|x_1\| \leqslant \|y\|$. Denote by (y_1, t_1) the unique point that satisfies the conditions: $\|y_1\| = \|x_1\|$, $s_1 = t_1$, y and y_1 are collinear. Note that $| \, \|y\| - \|y_1\| \, | \leqslant | \, \|y\| - \|x_1\| \, | \leqslant 2$ and therefore $\|(y_1, s_1) - (x_1, t_1))\| \leqslant 2$. Then

$$d(g(y_1, s_1), g(x_1, t_1)) \leqslant \left| \frac{\|y_1\|}{\|y_1\| + s_1} \|x_1 - y_1\| \right| \leqslant 2.$$

Let us estimate $d(g(y_1, s_1), g(y, s))$. Put

$$C = \sup\{d(g(x, t), g(y, s)) \mid \|(x, t) - (y, s)\| \leqslant 2, \|(y, s)\| \leqslant 1.$$

Since g is continuous, we see that $C < \infty$. Then

$$\begin{aligned}
&d(g(y_1, s_1), g(y, s)) \\
&= d\left(\frac{s_1}{\|y_1\| + s_1}\delta_{y_1} + \frac{\|y_1\|}{\|y_1\| + s_1}\delta_{\|y_1\|+s_1}, \frac{s}{\|y\| + s}\delta_y + \frac{\|y\|}{\|y\| + s}\delta_{\|y\|+s}\right) \\
&= \sup\left\{\left|\frac{s_1\alpha(y_1) - s\alpha(y)}{\|y\| + s}\right| \mid \alpha \in \text{Lip}(\mathbb{R}^n \vee \mathbb{R}_+), \ \alpha(\|y\| + s) = 0\right\} \\
&\leqslant \left|\frac{2(\|y_1\| + s}{\|y\| + s}\right| \leqslant 3 + C.
\end{aligned}$$

Finally, we obtain

$$\begin{aligned} d(g(x,t),(y,s)) \leqslant & d(g(x,t),g(x_1,t_1)) + d(g(x_1,t_1),g(y_1,s_1) + d(g(y_1,s_1),g(y,s)) \\ \leqslant & 1+2+3+C = 6+C. \end{aligned}$$

Since $\mathbb{R}_+^{n+1}$ is a geodesic metric space, it follows from Proposition 1.4 from [D] that the map g is asymptotically Lipschitz.

5. Remarks. The example from Section 4 demonstrates that the notion of a cone needs a slight modification in order to be more closely related to the "join with $\mathbb{R}_+$" construction. Namely, given a metric space (X,d), define its *modified cone* $\tilde{C}X$ as follows. As a set, $\tilde{C}X$ coincides with CX. The metric on $\tilde{C}X$ is the maximal metric $\varrho' \leqslant \varrho$ satisfying the following condition:

$$\varrho'([x_1,-d(x_1,x_0)],[x_2,-d(x_2,x_0)]) = |d(x_1,x_0) - d(x_2,x_0)|$$

for all $x_1, x_2 \in X$.

In a forthcoming publication we are going to consider some relations between $\tilde{C}X$ and CX.

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АСИМПТОТИЧНА КАТЕГОРІЯ І ПРОСТОРИ ЙМОВІРНІСНИХ МІР

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Наведено приклад власного метричного простору, простір ймовірнісних мір якого наділений метрикою Канторовича, не є абсолютним екстензором для асимптотичної категорії в сенсі Дранішникова.

Ключові слова: асимптотична категорія, ймовірнісні міри.

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